Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.

2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.

3. Only materials provided by the instructor can be present on the table during the exam.

4. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.

5. Only answers supported by work will be considered. Unsupported guesses will not be graded.

6. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.

7. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.

8. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

The test is 150 minutes, GOOD LUCK, and you may begin now!

<table>
<thead>
<tr>
<th>Question</th>
<th>Total Marks</th>
<th>Marks Obtained</th>
<th>Comments</th>
</tr>
</thead>
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<td>1</td>
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<td>2</td>
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<td>3</td>
<td>4+2=6</td>
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<td>6</td>
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<td>7</td>
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<tr>
<td>6</td>
<td>3+3=6</td>
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<tr>
<td>7</td>
<td>4+1=5</td>
<td></td>
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<tr>
<td>8</td>
<td>4+1=5</td>
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<tr>
<td>9</td>
<td>4+1=5</td>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>4+1=5</td>
<td></td>
<td></td>
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<tr>
<td>Total</td>
<td>55</td>
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<td></td>
</tr>
</tbody>
</table>
Extra blank page
1. (5 points) A portfolio contains 98 independent policies with benefit amount $B = 1$ each. For each policy the probability $q$ of a claim is $1/7$. Let $S$ be the total claims for the portfolio. Using a normal approximation, estimate $Pr(S > 10)$.
Work shown (6 points):

Hence the answer is (___)

2. (5 points) Calculate the variance for a 5000 benefit, 30 year endowment insurance providing the death benefit at the moment of death of a male age 35 at issue of the policy. Use the illustrative Life Table, the uniform distribution of deaths over each year of age assumption, and $i = 0.06$.

Work Shown

Hence the answer is (___)
3. (1+4=5 points) For a fully discrete whole life insurance of 1000 on (30):

(i) Mortality follows the Illustrative Life Table

(ii) \( i = 0.06 \)

(ii) \( \pi \) denotes the annual premium and \( L(\pi) \) denotes the loss-at-issue random variable for this insurance

Calculate the lowest premium \( \pi' \) such that the probability is less than 0.5 that the loss \( L(\pi') \) is positive.

Work Shown

Hence the answer is (___)

4. (4+2 = 6 marks) If \( P = 0.037736 \) and \( kq_x = c(0.96)^{k+1} \)

\[ i = 0.06. \]

Calculate Var(\( L \)) and \( 2V_x \).

Work Shown

Hence the answer is (___)
Directions: For questions 5 and 6 below, on the basis of the Illustrative Life Table with interest at 6%, consider a 5-year **term** life insurance of 1000 is issued on a fully discrete basis to each member of a group of $l_{50}$ persons at age 50.

5. $(14 \times 0.5 = 7$ marks) Trace the cash flow expected for this group and, as a by-product, obtain the benefit reserves.

Part of the work is already shown below. **Complete** the work by filling in the blanks with the correct numbers.

<table>
<thead>
<tr>
<th>Yr</th>
<th>Expected Benefit Premiums at Start of Year</th>
<th>Expected Fund at Start of Year</th>
<th>Expected Interest amount</th>
<th>Expected Death Claims</th>
<th>Expected Fund at Year End</th>
<th>Expected Number of Survivors at Year End</th>
<th>$1000 \times {V}_{50}^{\overline{5}\rvert}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35 214</td>
<td>529 884</td>
<td>92 233</td>
<td>1.04</td>
<td>1.04</td>
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<tr>
<td>2</td>
<td>40 540</td>
<td>571 432</td>
<td>88 407.68</td>
<td>1.64</td>
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<tr>
<td>3</td>
<td>724 452</td>
<td>616 416</td>
<td>87 791.26</td>
<td>1.21</td>
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<td>1.21</td>
</tr>
<tr>
<td>4</td>
<td>575 640</td>
<td>665 065</td>
<td>87 126.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>676 987</td>
<td>717 606</td>
<td>86 408.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. (3+3=6 points) Consider an insured who has survived to the end of the third policy year. For this insured, evaluate

a. $\text{Var} [\tilde{\mathcal{L}} | K(50) \geq 3]$ directly

b. $\text{Var} [\tilde{\mathcal{L}} | K(50) \geq 3]$ by means of the Hattendorf theorem
7. (4+1=5 points) You are given:

\[ \mu(x) = \begin{cases} 
0.05 & 50 \leq x \leq 60 \\
0.04 & 60 \leq x \leq 70 
\end{cases} \]

Calculate \( \frac{12}{4} q_{50} \)

a) 0.0828 

b) 0.2400 

c) 0.3416 

d) 0.4709 

e) 0.7866 

Work Shown (4 points)

Hence the answer is (___)

8. (4+1=5 marks) For a fully discrete 3-year endowment insurance of 1000 on \((x)\):

(i) \( i = 0.06 \)

(ii) \( q_x = q_{x+1} = 0.20 \)

(ii) \( 1000P_{x:3\mid} = 373.63 \)

(iv) \( hV \) is the benefit reserve of the policy at the end of year \( h \).

Calculate \( 2V - 1V \).

a) 320 

b) 325 

c) 330 

d) 335 

e) 340 

Work Shown (4 points)

Hence the answer is (___)
(4+1=5 marks) For a special 3-year term life insurance on (50), you are given:

(i) The death benefit of 10,000 is paid at the end of the year of death
(ii) The annual effective rate of interest is 4%
(iii) The benefit premium in year 1 is $10000A_{\overline{50}}\bar{\eta}$
(iv) The benefit premiums in years 2 and 3 are equal
(v) The mortality table has the following values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.05</td>
</tr>
<tr>
<td>51</td>
<td>0.06</td>
</tr>
<tr>
<td>52</td>
<td>0.07</td>
</tr>
<tr>
<td>53</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Calculate the benefit reserve at the end of year 2.

a) 0
b) 48.56
c) 50.51
d) 52.52
e) 53.16

Work Shown (4 points)

Hence the answer is (___)
10. (4+1=5 marks) For a special fully discrete 20-year endowment insurance on (40):

(i) The death benefit is 1000 for the first 10 years and 2000 thereafter. The pure endowment benefit is 2000.

(ii) The annual benefit *premium* is 40 for each of the first 10 years and 100 for each year thereafter.

(iii) $q_{40+k} = 0.001k + 0.001$, \(k = 8, 9, \ldots, 13\)

(iv) $i = 0.05$

(v) $\bar{a}_{51:m} = 7.1$

Calculate the benefit reserve at the end of year 10.

a) 490

b) 500

c) 530

d) 550

e) 560

Work Shown (4 points)

Hence the answer is (___)

END OF TEST PAPER