KFUPM | Term 131 | Name: ______ Sr ___
MATH 101-7 | Quiz 4 | ID: Solutions ______ Sec ___

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① (5 Points) Find the slope of the tangent line to the graph of $y = \dfrac{\tan x}{1+\cos x}$ at $x = \pi/3$.

The slope at any point $= y' = \dfrac{(1+\cos x)\sec^2 x + \tan x \sin x}{(1+\cos x)^2}$

$$\Rightarrow y'\Big|_{x=\frac{\pi}{3}} = \frac{(1+\frac{1}{2})(4) + (\sqrt{3})(\frac{\sqrt{3}}{2})}{(1+\frac{1}{2})^2}$$

$$= \frac{6+\frac{3}{2}}{\frac{9}{4}} = \frac{24+6}{9} = \frac{30}{9}$$

$= \frac{10}{3}$ = The required slope

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② (5 points) If $y = \sin^4(\csc^2(5x))$, find y'

We apply the Chain Rule:

$$y' = 4\sin^3(\csc^2(5x)) \cdot \cos(\csc^2(5x))(2\csc(5x)) \cdot (-\csc(5x)\cot(5x)) \cdot (5)$$

$$= -40\sin^3(\csc^2(5x)) \cdot \cos(\csc^2(5x)) \cdot \csc^2(5x) \cdot \cot(5x).$$

P.T.O →

③ (4 Points) If $y = \left(3x^2 + \sqrt{x^2-1}\right)^{5/2}$, find y'.

We apply the chain Rule:

$$y' = \frac{5}{2}\left(3x^2 + \sqrt{x^2-1}\right)^{3/2}\left(6x + \frac{2x}{2\sqrt{x^2-1}}\right)$$

$$= 5\left(3x^2 + \sqrt{x^2-1}\right)^{3/2}\left(3x + \frac{x}{\sqrt{x^2-1}}\right).$$

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④ (6 Points) If $xy + y^2 = 2$, then use implicit differentiation to find the value of y'' at $y = 1$

$$\boxed{\begin{array}{l} y = 1 \Rightarrow \\ x + 1 = 2 \Rightarrow \\ x = 1 \end{array}}$$

$$\frac{d}{dx}\left(xy + y^2\right) = \frac{d}{dx}(2) \Rightarrow$$

$$* \ \boxed{xy' + y + 2yy' = 0} \quad \text{at } x=1,\ y=1 \Rightarrow$$

$$y' + 1 + 2y' = 0 \Rightarrow \quad \boxed{y'\Big|_{x=1, y=1} = -\frac{1}{3}}$$

$$\boxed{*} \Rightarrow \frac{d}{dx}\left(xy' + y + 2yy'\right) = 0 \Rightarrow$$

$$xy'' + y' + y' + 2yy'' + 2(y')^2 = 0 \Rightarrow$$

when $x = 1$, $y = 1$, and $y' = -\frac{1}{3} \Rightarrow$

$$y'' - \frac{2}{3} + 2y'' + \frac{2}{9} = 0 \Rightarrow 3y'' = \frac{4}{9} \Rightarrow$$

$$y'' = \frac{4}{27}.$$
