Math 101 Quiz 1 10/09/2013

Ex1. Evaluate the following limits

a) $\lim\limits_{t \to -1} \dfrac{t^2+3t+2}{t^2-t-2}$, b) $\lim\limits_{x \to -1} \dfrac{\sqrt{x^2+8}-3}{x+1}$

Ex2 Find the slope of the tangent line to the curve $y = \dfrac{1}{x}$ at $x = \dfrac{1}{2}$

Solution

Ex1. a) $\lim\limits_{t \to -1} \dfrac{t^2+3t+2}{t^2-t-2} = \lim\limits_{t \to -1} \dfrac{(t+1)(t+2)}{(t+1)(t-2)} = \lim\limits_{t \to -1} \dfrac{t+2}{t-2} = -\dfrac{1}{3}$

b) $\lim\limits_{x \to -1} \dfrac{\sqrt{x^2+8}-3}{x+1} = \lim\limits_{x \to -1} \dfrac{(\sqrt{x^2+8}-3)(\sqrt{x^2+8}+3)}{(x+1)(\sqrt{x^2+8}+3)}$

$= \lim\limits_{x \to -1} \dfrac{x^2+8-9}{(x+1)(\sqrt{x^2+8}+3)} = \lim\limits_{x \to -1} \dfrac{x^2-1}{(x+1)(\sqrt{x^2+8}+3)}$

$= \lim\limits_{x \to -1} \dfrac{x-1}{\sqrt{x^2+8}+3} = \dfrac{-2}{6} = -\dfrac{1}{3}$.

Ex2. $\lim\limits_{h \to 0} \dfrac{\frac{1}{\frac{1}{2}+h} - \frac{1}{\frac{1}{2}}}{h} = \lim\limits_{h \to 0} \dfrac{\frac{2}{1+2h} - 2}{h}$

$= \lim\limits_{h \to 0} \dfrac{2-2(1+2h)}{h(1+2h)} = \lim\limits_{h \to 0} \dfrac{-4h}{h(1+2h)} = \lim\limits_{h \to 0} \dfrac{-4}{1+2h} = -4$

Hence the slope of the tangent line to the curve $y = \dfrac{1}{x}$ at $x = \dfrac{1}{2}$ is -4.