1. If \( f(x) = x^2 e^x \), then \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \frac{f'(x)}{h} \) = \( x^2 e^x + e^x \cdot 2x \) = \( e^x (x^2 + 2x) \)

a) \( e^x (x^2 + 2x) \)
b) \( 2xe^x \)
c) \( 2xh e^x \)
d) \( 2x^2 - x \)
e) \( e^x (x + 2) \)

2. If \( y = \frac{1}{2 - \sqrt{x}} \), then \( \frac{dy}{dx} = \frac{-1}{2\sqrt{x} (2 - \sqrt{x})^2} \) = \( \frac{1}{2\sqrt{x} (2 - \sqrt{x})^2} \)

a) \( \frac{1}{2\sqrt{x} (2 - \sqrt{x})^2} \)
b) \( \frac{-1}{(2 - \sqrt{x})^2} \)
c) \( \frac{-2}{\sqrt{x} (2 - \sqrt{x})} \)
d) \( \frac{-1}{\sqrt{x} (2 - \sqrt{x})^2} \)
e) \( \frac{1}{2(2 - \sqrt{x})^2} \)
3. The equation of the tangent line to the curve \( y = 2 \tan \left( \frac{\pi x}{4} \right) \) at \( x = 1 \) is

\[ - \frac{\pi}{4} x = 1 \Rightarrow y = 2 \tan \left( \frac{\pi}{4} \right) = 2.4 \Rightarrow (1, 2) \]

a) \( y = \pi x + 2 - \pi \)

b) \( y = 3\pi x + 2 - 3\pi \)

c) \( y = x + \frac{\pi}{4} \)

d) \( y = \frac{\pi}{4} x + 2 - \frac{\pi}{4} \)

e) \( y = -\pi x + 2 + \pi \)

4. The number of points at which the curve \( y = x^3 - 3x^2 + 4 \) has tangent lines parallel to the line \( 3x + y = 2 \) is

\[ 3x + y = 2 \Rightarrow y = -3x + 2 \Rightarrow S \Rightarrow -3 \]

a) One

b) Two

\[ 3x^2 - 6x = -3 \]

c) Three

\[ x^2 - 2x = -1 \]

d) Four

\[ (x - 1)^2 = 0 \]

e) Zero

\[ x = 4 \]
5. If \( f(x) = \left( \frac{x}{5} - \frac{5}{x} \right)^5 \), then \( f'(x) = 5 \left( \frac{x}{5} - \frac{5}{x} \right)^4 \cdot \left( \frac{1}{5} + \frac{5}{x^2} \right) \)

\[
= \left( \frac{x}{5} - \frac{5}{x} \right)^4 \left( 1 + \frac{25}{x^2} \right)
\]

a) \( \left( \frac{x}{5} - \frac{5}{x} \right)^4 \left( 1 + \frac{25}{x^2} \right) \)
a)

b) \( 5 \left( \frac{x}{5} - \frac{5}{x} \right)^4 \)

b)

c) \( \frac{5}{x^2} \left( \frac{x}{5} - \frac{5}{x} \right)^4 \)

c)

d) \( \left( \frac{x}{5} - \frac{5}{x} \right)^4 \left( 5 - \frac{1}{x^2} \right) \)

d)

e) \( \left( \frac{x}{5} - \frac{5}{x} \right)^4 \left( 1 + 5x \right) \)

e)

6. The linearization of \( f(x) = e^{\tan^{-1}(3x)} \) at \( x = 0 \) is given by

\[
L(x) = f(0) + f'(0)(x-0)
\]

\[
= e^0 + \frac{d}{dx} e^{\tan^{-1}(3x)} \bigg|_{x=0} (x-0)
\]

\[
= 1 + \frac{3}{1+0} (x-0)
\]

\[
= 1 + 3x
\]

a) \( L(x) = 1 + 3x \)
a)

b) \( L(x) = 3x \)
b)

c) \( L(x) = 3-x \)
c)

d) \( L(x) = 2+x \)
d)

e) \( L(x) = 1-2x \)
e)
7. The position function of a body moving in a straight line is

\[ s(t) = t^3 - 6t^2 + 9t, \quad t \geq 0 \]

The body changes direction at \( v(t) = s'(t) = 0 \)

\[ 3t^2 - 12t + 9 = 0 \]
\[ t^2 - 4t + 3 = 0 \]
\[ (t-1)(t-3) = 0 \]

a) \( t = 1 \) and \( t = 3 \)

b) \( t = 2 \) and \( t = 5 \)

c) \( t = 3 \) only

d) \( t = 1 \) and \( t = 4 \)

e) the body never changes direction

8. If \( y = \log_2 (8t \ln t) \), then \( \frac{dy}{dt} = \)

a) \( \frac{1}{t} \)

b) \( t \)

c) \( 3 \ln t \)

d) \( \frac{1}{\ln t} \)

e) \( \log_2 t \)
9. If \( z = \sqrt{u(u + 1)} \) and \( u = \frac{x}{x - 1} \), then \( \frac{dz}{dx} \Big|_{x=2} = \)

\( \frac{d}{dx} z = \frac{d}{du} u \cdot \frac{du}{dx} \cdot \frac{dz}{du} \)

\( \Rightarrow \frac{z}{z-1} = 2 \)

- \( \frac{d}{dx} u = \frac{u}{u+1} \cdot (2u+1) \cdot \frac{z-1}{z} \)

\( \Rightarrow \frac{z}{z-1} = 2 \)

a) \( \frac{-5}{3\sqrt{3}} \)

b) \( \frac{1}{3\sqrt{3}} \)

c) \( \frac{4\sqrt{3}}{3} \)

d) \( \frac{-4}{3\sqrt{3}} \)

e) \( \frac{2}{\sqrt{3}} \)

10. The tangent line to the graph of the curve \( y = \ln \left( \frac{\sqrt{\tan(2x)}}{1 + \sec(2x)} \right) \) at \( x = \frac{\pi}{6} \) is

\( y = \frac{4}{\sqrt{3}} - \frac{2\sqrt{3}}{1 + 2} \)

- a horizontal line
- b) a vertical line
- c) with slope \( -\frac{\sqrt{3}}{3} \)
- d) with slope \( \sqrt{3} \)
- e) with slope \( 2\sqrt{3} \)
11. If \( x > 0 \), then \( \frac{d}{dx} \left[ \sin^{-1} \left( \frac{x - 4}{x + 4} \right) \right] = \frac{1}{\sqrt{1 - (\frac{x - 4}{x + 4})^2}} \cdot \frac{(x + 4) \cdot 1 - (x - 4) \cdot 1}{(x + 4)^2} \)

\[
\begin{align*}
\text{a)} & \quad \frac{2\sqrt{x}}{x(x + 4)} \\
\text{b)} & \quad \frac{8\sqrt{x}}{x + 4} \\
\text{c)} & \quad \frac{4\sqrt{x}}{x} \\
\text{d)} & \quad \frac{8}{1 + \sqrt{x}} \\
\text{e)} & \quad (x + 4)\sqrt{x}
\end{align*}
\]

12. The radius of a sphere was measured to be 20 cm with a possible error in measurement of at most 0.05 cm. The maximum error in the computed volume of the sphere is approximately equal to

\[ V = \frac{4}{3} \pi r^3 \quad , \quad r = 20 , \quad dr = 0.05 \]

\[
\begin{align*}
\text{a)} & \quad 80 \pi \\
\text{b)} & \quad 60 \pi \\
\text{c)} & \quad 40 \pi \\
\text{d)} & \quad 20 \pi \\
\text{e)} & \quad 10 \pi
\end{align*}
\]
13. If $5x^5 - y^5 = 1$, then $y'' =$

\[
\begin{align*}
\text{a) } & \frac{20x^3}{y^9} \\
\text{b) } & \frac{5x^3}{y^9} \\
\text{c) } & \frac{5x^4}{y^5} \\
\text{d) } & \frac{20x^4}{y^5} \\
\text{e) } & \frac{y^9 - 5x^4}{y^9} \\
\end{align*}
\]

\[
\Rightarrow y' = \frac{25x^4 - 5y^4}{y^9} \\
\Rightarrow y'' = 5 \cdot \frac{4 \cdot 4x^3 - x^4 \cdot 4y^4 \cdot \frac{5x^4}{y^9}}{y^8}
\]

\[
= 5 \cdot \frac{4x^3 - 20x^8}{y^8}
\]

\[
= 5 \cdot \frac{4x^3 - 20x^8}{y^8}
\]

\[
= 5 \cdot \frac{4x^3}{y^8}
\]

\[
= \frac{20x^3}{y^8}
\]

14. If $y = \frac{\cos x}{e^x}$ then $y'' + 2y' + 3y =$

\[
\begin{align*}
\text{a) } & \frac{\cos x}{e^x} \\
\text{b) } & -\sin x \\
\text{c) } & \frac{\sin x + \cos x}{e^x} \\
\text{d) } & \frac{\sin x}{e^{2x}} \\
\text{e) } & \frac{\sin x - \cos x}{e^{2x}} \\
\end{align*}
\]

\[
\Rightarrow y' = -\frac{\cos x}{e^x} - \frac{\sin x}{e^x}
\]

\[
\Rightarrow y'' = \left(-\frac{\cos x}{e^x} - \frac{\sin x}{e^x}\right) - \left(-\frac{\cos x}{e^x} - \frac{\sin x}{e^x}\right)
\]

\[
= 2 \frac{\cos x}{e^x}
\]

\[
= \frac{2 \cos x}{e^x}
\]

\[
\Rightarrow y'' + 2y' + 3y = 2 \frac{\cos x}{e^x} - 2 \frac{\cos x}{e^x} - 2 \frac{\cos x}{e^x}
\]

\[
+ 3 \frac{\cos x}{e^x} = \frac{\cos x}{e^x}
\]
15. Let \( f(x) = 1 + 2x - x^2, x \leq 1. \) Then \( \frac{df^{-1}}{dx} \bigg|_{x=2} = \frac{1}{\frac{df}{dx} \bigg|_{x=f^{-1}(2)}} \)

- a) \( \frac{1}{4} \)
- b) \( \frac{1}{2} \)
- c) \( -\frac{1}{4} \)
- d) \( -1 \)
- e) \( \frac{1}{3} \)

So the answer is \( \frac{1}{4} \).

16. If \( y = (\sin x)^{\sqrt{x}} \), then \( \frac{dy}{dx} = \) 

- a) \( \frac{3x \cot x + \ln(\sin x)}{3\sqrt{x}} \)
- b) \( \frac{x \cot x + 3 \ln(\sin x)}{3\sqrt{x}} \)
- c) \( \frac{x \tan x + \ln(\sin x)}{3\sqrt{x}} \)
- d) \( \frac{x \tan x - \ln(\sin x)}{3\sqrt{x}} \)
- e) \( \frac{x \cot x + \ln(\sin x)}{3\sqrt{x}} \)
17. If the normal line to the curve \( x^2 - xy + y^2 = 1 \) at \((1, 1)\) intersects the curve at another point \((a, b)\), then \( a + b = \)

\[ 2 \frac{dy}{dx} - y + 2y' = 0 \implies \frac{dy}{dx} = \frac{y}{2} \]

Slope of the normal line \( y = \frac{y}{x} \)

a) -2  
b) 2  
c) 0  
d) 3  
e) -1

18. A street light is mounted at the top of a 5-meter-tall pole. A man 2m tall walks away from the pole with a speed of \( \frac{3}{2} \) m/s along a straight path. How fast is his shadow moving when he is 10 m from the pole?

a) 1 m/s  
b) 2 m/s  
c) 3 m/s  
d) 4 m/s  
e) 5 m/s

\[ \frac{dx}{dt} = \frac{3}{2} \quad \frac{dy}{dt} = ? \quad \text{when} \quad x = 10 \]

\[ \frac{y}{2} = \frac{x+y}{5} \]

\[ \Rightarrow 5y = 2x + 2y \]

\[ \Rightarrow 3y = 2x \]

\[ \Rightarrow y = \frac{2}{3} x \]

\[ \Rightarrow \frac{dy}{dt} = \frac{2}{3} \frac{dx}{dt} \]

\[ = \frac{2}{3} \times \frac{3}{2} = 1 \]
19. If the line \( y = 2x + 8 \) is a tangent line to the curve \( y = \frac{c}{x+2} \), then \( c^2 - 3c + 4 = \)

Let \((\alpha, \beta)\) be the point of tangency. Then,

\[
\begin{align*}
\beta &= 2\alpha + 8 \\
\beta &= \frac{c}{\alpha+2}
\end{align*}
\]

\( \Rightarrow \) \( \alpha = \beta (\alpha+2) - (2\alpha+8)(\alpha+1) \)

\( \Rightarrow \alpha = -3, \quad \frac{-2}{\alpha+1} \) (reject)

So \( \alpha = -3 \Rightarrow \beta = -2(-3+1) = 2 \)

\( \Rightarrow \alpha = 3 \quad \beta = 3 + 4 = -8 + 6 + 2 = 0 \)

20. If the function

\( f(x) = \begin{cases} 
ax + 3 & \text{if } x \geq -1 \\
bx^2 + ax & \text{if } x < -1
\end{cases} \)

is differentiable on \((-\infty, \infty)\), then \( f(-2) = \)

- \( f \) has to be continuous at \( x = -1 : \)

\[
\lim_{{x \to -1^-}} f(x) = \lim_{{x \to -1^+}} f(x) = b + a = -a + 3 \quad \Rightarrow b + 2a = 3 \quad (1)
\]

- \( f \) has to be differentiable at \( x = -1 : \)

\( f \) has to be defined, at \( x = -1 = \) left derivative, at \( x = -1 = \) right derivative.

\[
a = -2b - a \quad \Rightarrow b = -a \quad (2)
\]

Solving \((1) \& (2)\), we get

\[
a = 3, \quad b = -3
\]

So \( f(-2) = b (-2)^2 - a (-2) = -3(4) - (3) (-2) = -12 + 6 = -6 \).