• KFUPM | Term 131 | Name: Solutions | Sr: ___

• MATH 102-8 | Quiz #3 | ID: ______ | Sec: ___

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① (8-points) Find the equation of the tangent line to the curve $y = \sinh(\frac{1}{2}x)$ at $x = \ln 9$

slope of the tangent at any point is $y' = \frac{1}{2}\cosh(\frac{1}{2}x) \Rightarrow$

$$y'\Big|_{x=\ln 9} = \frac{1}{2}\cosh(\tfrac{1}{2}\ln 9) = \frac{1}{2}\cosh(\ln 3)$$

$$= \frac{1}{2}\left[\frac{e^{\ln 3}+e^{-\ln 3}}{2}\right] = \frac{1}{4}\left[3+\tfrac{1}{3}\right] = \frac{10}{12} = \frac{5}{6}$$

And $x = \ln 9 \Rightarrow y = \sinh(\frac{1}{2}\ln 9) = \sinh(\ln 3)$

$$\Rightarrow y = \frac{1}{2}(e^{\ln 3} - e^{-\ln 3}) = \frac{1}{2}(3 - \tfrac{1}{3}) = \frac{8}{6} = \frac{4}{3}$$

$\Rightarrow$ Point of tangency is $(\ln 9, \frac{4}{3}) \Rightarrow$ The required equation is $y - \frac{4}{3} = \frac{5}{6}(x - \ln 9)$.

OR $y = \frac{5}{6}x + \frac{4}{3} - \frac{5}{6}\ln 9$.

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② (5-points) Evaluate the integral:

$\int x^3 \cosh(3x)\,dx$ [You may use tabular integration]

$$= \frac{1}{3}x^3\sinh(3x) - \frac{1}{3}x^2\cosh(3x) + \frac{2}{9}x\sinh(3x) - \frac{2}{27}\cosh(3x) + C$$

x^3		$\cosh(3x)$
$3x^2$	$+$	$\frac{1}{3}\sinh(3x)$
$6x$	$-$	$\frac{1}{9}\cosh(3x)$
6	$+$	$\frac{1}{27}\sinh(3x)$
0	$-$	$\frac{1}{81}\cosh(3x)$

P.T.O →

• 3 (7-points) Evaluate the integral

$$I = \int x(\ln 3x)^2\, dx$$

we use integration by parts:

$$u = (\ln 3x)^2 \quad , \quad dv = x\,dx$$

$$\Longrightarrow du = 2(\ln 3x) \cdot \frac{3dx}{3x} = \frac{2\ln 3x}{x} dx, \quad v = \frac{1}{2}x^2$$

$$\Longrightarrow I = \frac{1}{2}x^2(\ln 3x)^2 - \int x \ln 3x \, du \quad (i)$$

Let $I_1 = \int x \ln 3x\, dx$ (ii)

we use integration by parts once again

Let $u = \ln 3x \quad , \quad dv = x\,dx$

$$\Longrightarrow du = \frac{1}{3x}(3)\,dx = \frac{1}{x}dx \quad , \quad v = \frac{1}{2}x^2$$

$$\Longrightarrow I_1 = \frac{1}{2}x^2 \ln 3x - \frac{1}{2}\int x\, dx$$

$$= \frac{1}{2}x^2 \ln 3x - \frac{1}{4}x^2 \quad (iii)$$

(i), (ii) and (iii) $\Longrightarrow$

$$I = \frac{1}{2}x^2(\ln 3x)^2 - \frac{1}{2}x^2 \ln 3x + \frac{1}{4}x^2 + C.$$

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① (8-points) Find an equation of the tangent line to the curve $y = \cosh(\frac{1}{2}x)$ at $x = \ln 4$

Slope of the tangent at any point is $y' = \frac{1}{2}\sinh(\frac{1}{2}x)$

$$\Rightarrow y'\Big|_{x=\ln 4} = \frac{1}{2}\sinh\left(\frac{1}{2}\ln 4\right) = \frac{1}{2}\sinh(\ln 2) = \frac{1}{2}\left(\frac{e^{\ln 2} - e^{-\ln 2}}{2}\right)$$

$$= \frac{1}{4}\left(2 - \tfrac{1}{2}\right) = \frac{3}{8}.$$

And $x = \ln 4 \Longrightarrow y = \cosh(\frac{1}{2}\ln 4) = \cosh(\ln 2) = \dfrac{e^{\ln 2} + e^{-\ln 2}}{2}$

$$= \frac{1}{2}\left(2 + \tfrac{1}{2}\right) = \frac{5}{4}$$

$\Longrightarrow$ point of tangency is $(\ln 4, \frac{5}{4}) \Longrightarrow$ The required equation is $y - \frac{5}{4} = \frac{3}{8}(x - \ln 4)$

OR $\quad y = \frac{5}{8}x + \frac{5}{4} - \frac{3}{8}\ln 4.$

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② (5-points) Evaluate the integral:

$$\int x^3 \sinh(2x)\, dx \qquad \text{[You may use tabular Integration]}$$

$$= \frac{1}{2}x^3\cosh(2x) - \frac{3}{4}x^2\sinh(2x) + \frac{3}{4}x\cosh(2x) - \frac{3}{8}\sinh(2x) + C.$$

x^3	$+$	$\sinh(2x)$
$3x^2$	$-$	$\frac{1}{2}\cosh(2x)$
$6x$	$+$	$\frac{1}{4}\sinh(2x)$
6	$-$	$\frac{1}{8}\cosh(2x)$
0		$\frac{1}{16}\sinh(2x)$

P.T.O →

③ (7-points) Evaluate the integral

$$I = \int x (\ln 2x)^2 \, dx$$

We use integration by parts

Let $u = (\ln 2x)^2$, $v = x\,dx$

$\Longrightarrow$ $du = 2(\ln 2x)\frac{2\,dx}{2x} = \frac{2\ln 2x}{x}dx$, $v = \frac{1}{2}x^2$

$\Longrightarrow$ $I = \frac{1}{2}x^2(\ln 2x)^2 - \int x \ln 2x \, dx$ (i)

Let $I_1 = \int x \ln 2x \, dx$ (ii)

and apply integration by parts once more:

Let $u = \ln 2x$, $dv = x\,dx$

$\Longrightarrow$ $du = \frac{1}{2x}(2)dx = \frac{1}{x}dx$, $v = \frac{1}{2}x^2$

$\Longrightarrow$ $I_1 = \frac{1}{2}x^2 \ln 2x - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2 \ln 2x - \frac{1}{4}x^2$ (iii)

(i), (ii), and (iii) $\Longrightarrow$

$$I = \frac{1}{2}x^2(\ln 2x)^2 - \frac{1}{2}x^2 \ln 2x + \frac{1}{4}x^2 + C.$$