King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201
Exam II
Term 131
Monday 25/11/2013
Net Time Allowed: 120 minutes

MASTER VERSION

Key

(10 pts) 1. Find an equation for the plane passing through the point $Q(-1,-4,5)$ and containing the line with parametric equations

$$x = 1 - t,\ y = 2t - 3,\ z = t.$$

The line contains the point $P(1,-3,0)$ (1pt)

and has direction vector $\vec{u} = \langle -1, 2, 1 \rangle$ (1pt)

A normal vector $\vec{n}$ to the plane is given by

$\vec{n} = \overrightarrow{PQ} \times \vec{u}$ (2pts) $\quad \overrightarrow{PQ} = \langle -2, -1, 5 \rangle$ (2pts)

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 5 \\ -1 & 2 & 1 \end{vmatrix} = \langle -11, -3, -5 \rangle \text{ (2pts)}$$

Thus an equation of the plane is given by

$$-11(x-1) - 3(y+3) - 5(z-0) = 0 \quad \text{(2pts)}$$

$$\Leftrightarrow \quad -11x - 3y - 5z = -2$$

$$\Leftrightarrow \quad 11x + 3y + 5z = 2$$

(5 pts) 2. A) Find $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation

$$\sin(xyz) = x + 2y + 3z$$

Let $F(x,y,z) = \sin(xyz) - x - 2y - 3z$ (1 pt)

$$\frac{\partial z}{\partial y} = \underbrace{\frac{-F_y}{F_z}}_{(2pts)} = -\frac{xz\cos(xyz) - 2}{xy\cos(xyz) - 3}$$

$$= \frac{2 - xz\cos(xyz)}{xy\cos(xyz) - 3}$$ (2 pts)

(5 pts) B) Give an equation for the tangent plane to the surface $x^2 + y^3 + z^2 = 0$ at the point $(2, -2, 2)$.

Let $F(x,y,z) = x^2 + y^3 + z^2$. The equation of the tangent line is given by

$$\underbrace{F_x(2,-2,2)(x-2) + F_y(2,-2,2)(y+2) + F_z(2,-2,2)(z-2) = 0}_{(1 pt)}$$

$F_x = 2x$, $F_y = 3y^2$, $F_z = 2z$

$\underbrace{F_x(2,-2,2) = 4}_{(1pt)}$, $\underbrace{F_y(2,-2,2) = 12}_{(1pt)}$, $\underbrace{F_z(2,-2,2) = 4}_{(1pt)}$

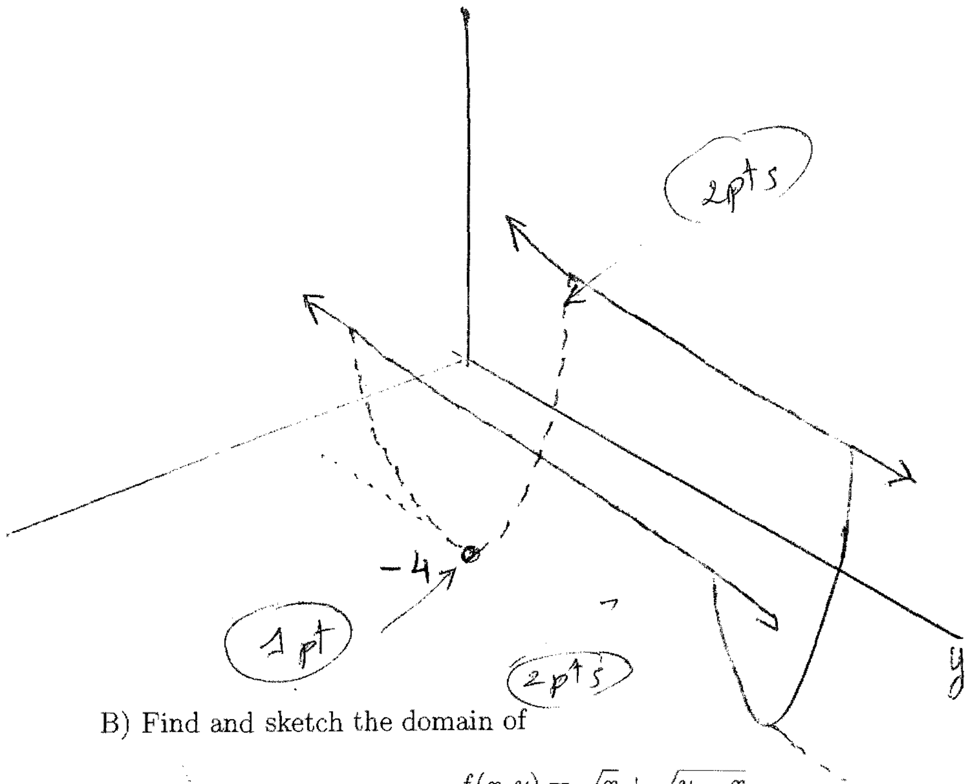
the equation of the tangent line is

$$4(x-2) + 12(y+2) + 4(z-2) = 0 \iff$$

$$x + 3y + z = -2$$ (1 pt)

3. A) Sketch the surface given by $z = x^2 - 4$

(5 pts)

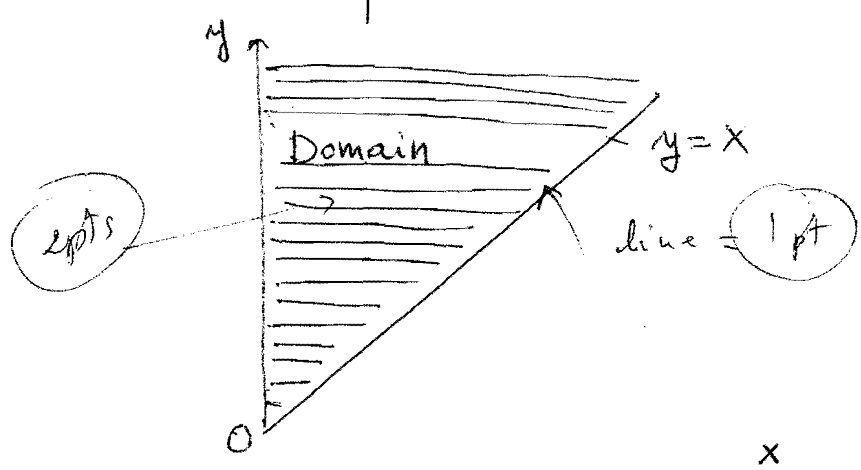


(5 pts)

B) Find and sketch the domain of

$$f(x,y) = \sqrt{x} + \sqrt{y-x}$$

$$\left.\begin{aligned} \text{Domain} &= \{(x,y) \mid x \geq 0,\ y-x \geq 0\} \\ &= \{(x,y) \mid x \geq 0,\ y \geq x\} \end{aligned}\right\} \text{2pts}$$



(10 pts)

4. Let $f = f(x, y)$ such that $(D_{\vec{j}} f)|_{(1,1)} = -\sqrt{2}$ and $(D_{\vec{u}} f)|_{(1,1)} = 3$ where $\vec{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$. Find $(D_{\vec{v}} f)|_{(1,1)}$ if $\vec{v} = \frac{1}{\sqrt{3}}\langle 1, \sqrt{2} \rangle$.

(1 pt) $f_y(1,1) = -\sqrt{2}$ and $\Leftarrow D_{\vec{j}} f|_{(1,1)} = -\sqrt{2}$

$\Leftarrow (D_{\vec{u}} f)|_{(1,1)} = 3$

(2 pts) $f_x(1,1) \cdot \frac{1}{\sqrt{2}} + f_y(1,1) \frac{1}{\sqrt{2}} = 3 \iff$

$f_x(1,1) \frac{1}{\sqrt{2}} - 1 = 3 \iff$

(3 pts) $f_x(1,1) = 4\sqrt{2}$

Thus

(2 pts) $(D_{\vec{v}} f)(1,1) = f_x(1,1) \frac{1}{\sqrt{3}} + f_y(1,1) \frac{\sqrt{2}}{\sqrt{3}}$

$= \frac{4\sqrt{2}}{\sqrt{3}} - \frac{2}{\sqrt{3}}$

(2 pts) $= \frac{4\sqrt{2} - 2}{\sqrt{3}}$

5. Consider the function

$$f(x,y) = 3x^2 - xy + y^3$$

(5 pts)

A) In what direction (unit vector) does f decrease most rapidly at $(-1,1)$?

$$\nabla f(-1,1) = \langle f_x(-1,1),\ f_y(-1,1) \rangle \quad \text{1 pt}$$

$$= \langle -7, 4 \rangle \quad \text{1 pt}$$

The direction of rapid decrease is given by

$$\underbrace{-\frac{\nabla f(-1,1)}{|\nabla f(-1,1)|}}_{\text{2 pts}} = -\frac{1}{\sqrt{65}} \langle -7, 4 \rangle$$

$$= \left\langle \frac{7}{\sqrt{65}}, \frac{-4}{\sqrt{65}} \right\rangle \quad \text{1 pt}$$

B) In what direction is the rate of change of f at $(-1,1)$ equal to zero. (Find all directions).

The directions of zero change are

3 pts $\quad \vec{u}_1 = \left\langle \frac{4}{\sqrt{65}}, \frac{7}{\sqrt{65}} \right\rangle$ and

2 pts $\quad \vec{u}_2 = -\vec{u}_1 = \left\langle \frac{-4}{\sqrt{65}}, \frac{-7}{\sqrt{65}} \right\rangle$

Q	MM	V1	V2	V3	V4
1	a	a	d	c	e
2	a	e	a	c	e
3	a	e	e	a	a
4	a	e	a	a	e
5	a	e	d	b	c
6	a	a	e	d	c
7	a	d	b	d	d
8	a	a	e	b	a
9	a	a	a	a	e
10	a	d	a	d	b