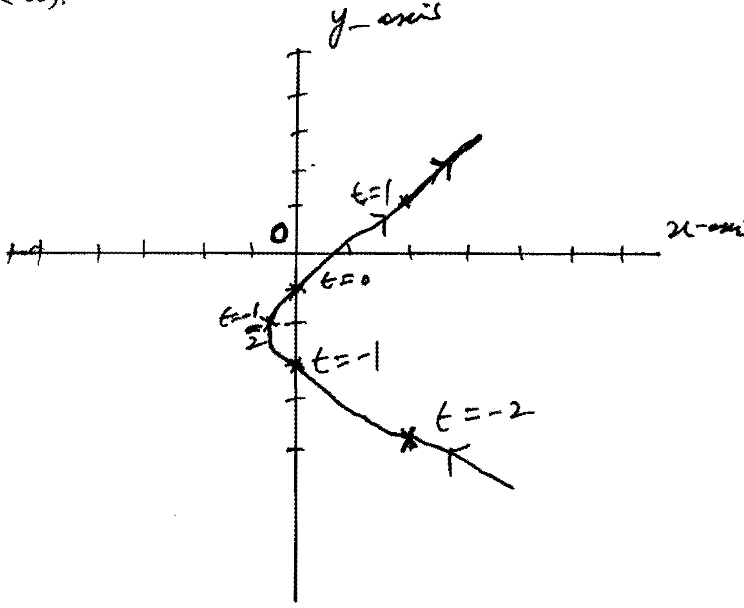
KFUPM Term (131) Name____________ Serial#________

MATH 201-[Sections-18, 22] Quiz # 1(a) ID# "Solution" Section_____

Time: 20 Minutes Date: 26-9-2013

1) Sketch the following parametric curve and identify particle's path on it:
$x = t^2 + t, y = 2t - 1\ (-\infty < t < \infty)$.

t	$x=t^2+t$	$y=2t-1$
-2	2	-5
-1	0	-3
$-\frac{1}{2}$	$-\frac{1}{4}$	-2
0	0	-1
1	2	1



2) Find slope of the curve: $x \sin t + 2x = t$, —(1) $t \sin t - 2t = y$ —(2) at $t = \pi$.

By (1), on differentiating w.r.t. "t" we get: $\frac{dx}{dt}\sin t + x\cos t + 2\frac{dx}{dt} = 1$

$\frac{dx}{dt}(\sin t + 2) = 1 - x\cos t$

$\frac{dx}{dt} = \frac{1 - x\cos t}{\sin t + 2}$

At $t=\pi$, By (1) we get: $x\sin\pi + 2x = \pi$

$\Rightarrow 0 + 2x = \pi$

$\Rightarrow 2x = \pi \Rightarrow x = \frac{\pi}{2}$ ——— (*)

By (2), on differentiating w.r.t. "t" we get: $t\cos t + \sin t - 2 = \frac{dy}{dt}$.

$$\text{slope} = \left.\frac{dy}{dx}\right|_{t=\pi} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\pi} \quad \frac{t\cos t + \sin t - 2\Big|_{t=\pi}}{\left(\frac{1 - x\cos t}{\sin t + 2}\right)\Big|_{t=\pi}}$$

$$= \frac{\pi\cos\pi + \sin\pi - 2}{\left(\frac{1 - \left(\frac{\pi}{2}\right)\cos\pi}{\sin\pi + 2}\right)} \qquad \text{By } (*);\ x = \frac{\pi}{2}$$

$$= \frac{-\pi - 2}{\frac{\left(1 + \pi/2\right)}{2}} = \frac{-\pi - 2}{\frac{2+\pi}{4}} = \frac{-4\pi - 8}{2+\pi} = \frac{4 \times \frac{22}{7} - 8}{2 + \frac{22}{7}} = -\frac{144}{36} = -4$$

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MATH 201-[Sections-18, 22] Quiz # 1(b) ID# "Solution" Section____

Time: 20 Minutes Date: 26-9-2013

1) Write $r\sin\left(\theta+\frac{\pi}{6}\right)=3$ in Cartesian Coordinates and then find slope of the resulting curve.

$$r\left[\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right]=3$$

$$r\sin\theta\left(\frac{\sqrt{3}}{2}\right)+r\cos\theta\left(\frac{1}{2}\right)=3$$

$$y\left(\frac{\sqrt{3}}{2}\right)+x\left(\frac{1}{2}\right)=3$$

$$\frac{\sqrt{3}}{2}y=3-\frac{x}{2}$$

$$y=\frac{2}{\sqrt{3}}\left(3-\frac{x}{2}\right)=2\sqrt{3}-\frac{x}{\sqrt{3}}=\frac{-1x}{\sqrt{3}}+2\sqrt{2}$$

$$y=-\frac{1}{\sqrt{3}}x+2\sqrt{3}\quad(\text{line})$$

$\Rightarrow$ slope of this line is $-\frac{1}{\sqrt{3}}$.

2) Find area A of the surface generated by revolving the curve $x=\cos^2 t,\ y=\sin^2 t\ (0\le t\le \pi/2)$ about the x-axis.

$$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=\sqrt{(-2\cos t\sin t)^2+(2\sin t\cos t)^2}$$

$$=\sqrt{4\cos^2 t\sin^2 t+4\sin^2 t\cos^2 t}$$

$$=2\sqrt{2}\sin t\cos t$$

$$A=\int_0^{\pi/2}2\pi\sin^2 t\,\left[2\sqrt{2}\sin t\cos t\right]dt$$

$$=4\sqrt{2}\pi\int_0^{\pi/2}\sin^3 t\cos t\,dt$$

$u=\sin t$

$du=\cos t\,dt.$

$$=\cancel{4}\sqrt{2}\pi\left[\frac{\sin^4 t}{\cancel{4}}\right]_0^{\pi/2}$$

$$=\sqrt{2}\pi[1-0]=\sqrt{2}\pi$$

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MATH 201-[Sections-18, 22] Quiz # 1(c) ID# "Solution" Section______

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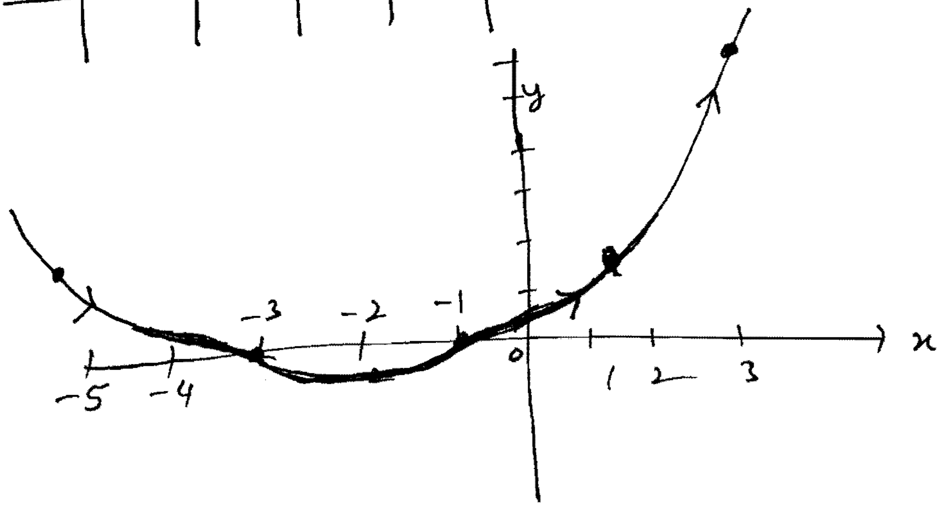
1) Sketch the parametric curve $C: x = 2t - 1, y = t + t^2$. Indicate by an arrow how the graph is traced as t increases.

t	-2	-1	0	1	2
x	-5	-3	-1	1	3
y	2	0	0	2	6



2) Write the polar equation $r = 2\cos\theta + 2\sin\theta$ in Cartesian Coordinates and sketch the graph of the resulting equation. Through which points of the axes the graph will pass?

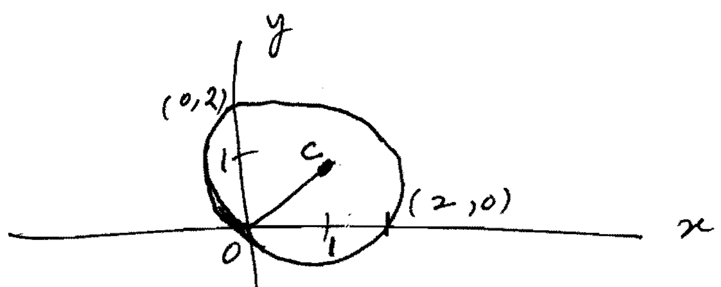
$$r^2 = 2r\cos\theta + 2r\sin\theta$$

$$x^2 + y^2 = 2x + 2y$$

$$x^2 - 2x^2 + y^2 - 2y = 0$$

$$(x-1)^2 + (y-1)^2 = 2 = (\sqrt{2})^2$$

Equation of a circle with centre C(1,1) & radius $= \sqrt{2}$



It passes the points (0,0), (2,0) & (0,2) on x-axis and y-axis.