

Full Name:

ID:

Serial number:

Question 1. Use the existence and uniqueness theorem to find the largest interval of definition  $I$  such that the IVP

$$\begin{cases} (\cot x)y'' + (\cos x)y & = 4x \\ y(2) = 0 \quad \text{and} \quad y'(2) & = 1 \end{cases}$$

has a unique solution.

Question 2. Determine whether the functions  $f_1(x) = e^x$ ,  $f_2(x) = \sinh x$  and  $f_3(x) = \cosh x$  are linearly independent or linearly dependent on  $(-\infty, \infty)$ .

Question 3. Given that  $y_1 = x$ ,  $y_2 = x^2$  and  $y_3 = x^2 \ln x$  are solutions of the homogeneous DE:

$$x^3 y''' - 2x^2 y'' + 4xy' - 4y = 0 \quad \text{on} \quad (0, \infty).$$

Find the general solution of this DE (justify your answer).

Question 4. The function  $y_1 = x$  is a solution of  $x^2 y'' + xy' - y = 0$  on  $(0, \infty)$ . Use Reduction of Order method to find the general solution of this DE.