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MATH 202.6 (Term 131)

Quiz 1 (Sects. 1.1-1.2, 2.2-2.3) **Duration: 20mn**

Name: **ID number:**

1.)(3pts)Do the following IVP have unique solutions?

a.) $\begin{cases} y' = \frac{(x^2+1)\ln y}{1-y} \\ y(0) = 0, \end{cases}$ **b.)** $\begin{cases} y' = \frac{(x^2+1)\ln y}{1-y} \\ y(0) = 2. \end{cases}$

2.)(4pts)Solve the DE: $e^{-2x}dy = (y-y^2)xdx$.

3.)(3pts)Solve the DE: $dx = (\sin 2y + x\tan y)dy$.

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1) a) $f(x,y) = \frac{(x^2+1)\ln y}{1-y}$, $y>0$, $y \neq 1$

$\frac{\partial f}{\partial y}(x,y) = (x^2+1)\left[\frac{\frac{1}{y}(1-y) - \ln y}{(1-y)^2}\right]$, $y>0$, $y\neq 1$

f and $\frac{\partial f}{\partial y}$ are not continuous at $(0,0)$

Thus, we can't decide whether the IVP has unique solution or not

b) f and $\frac{\partial f}{\partial y}$ are continuous at $(0,2)$, So the IVP has a unique solution

2) $e^{-2x}dy = (y-y^2)x\,dx$

$\int \frac{dy}{y-y^2} = \int e^{2x}x\,dx$

$\frac{1}{y(1-y)} = \frac{a}{y} + \frac{b}{1-y}$ $\Big|$ $a=1$, $b=1$

$\int \frac{dy}{y(1-y)} = \int\left(\frac{1}{y} + \frac{1}{1-y}\right)dy$

$= \ln|y| - \ln|1-y|$

$= \ln\left|\frac{y}{1-y}\right|$

We integrate by part $\int xe^{2x}dx$.

$u' = e^{2x}$, $u = \frac{e^{2x}}{2}$

$v = x$, $v' = 1$

$\int xe^{2x}dx = x\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}dx$

$= \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x}$

So, $\boxed{\ln\left|\frac{y}{1-y}\right| = \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + C}$

3) $dx = (\sin 2y + x\tan y)\,dy$

$\frac{dx}{dy} - \tan y\, x = \sin 2y$

This is a linear DE in x.

An integrating factor $e^{-\int \tan y\, dy}$

$= e^{\ln|\cos y|} = \cos y$, $\cos y\, x$

$\Rightarrow \frac{d}{dy}(x\cos y) = \sin 2y \cos y$

$= 2\cos^2 y \sin y$

$\Rightarrow x\cos y = \int 2\cos^2 y \sin y\, dy$

$\boxed{x\cos y = -\frac{2}{3}\cos^3 y + C}$

MATH 202.9 (Term 131)

Quiz 1 (Sects. 1.1-1.2, 2.2-2.3) Duration: 20mn

Name: **ID number:**

1.)(3pts)Do the following IVP have unique solutions?

a.) $\begin{cases} y' = \frac{x\sqrt{4-y^2}}{y} \\ y(0) = 0, \end{cases}$ **b.)** $\begin{cases} y' = \frac{x\sqrt{4-y^2}}{y} \\ y(0) = 1. \end{cases}$

2.)(4pts)Solve the DE: $(y^2+1)dx = y\sec^2 x dy$.

3.)(3pts)Solve the DE: $\frac{1}{x}\frac{dy}{dx} - 2y = \frac{2x+1}{x}$.

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1) a) $f(x,y) = \frac{x\sqrt{4-y^2}}{y}$, $y \in [-2,2]$, $y \neq 0$

$\frac{\partial f}{\partial y}(x,y) = x\left[\frac{-y^2}{y^2\sqrt{4-y^2}} - \frac{\sqrt{4-y^2}}{y^2}\right]$

f and $\frac{\partial f}{\partial y}$ are not continuous at $(0,0)$

So, we cannot decide whether the IVP has unique solution or not.

b) f and $\frac{\partial f}{\partial y}$ are continuous at $(0,1)$, so the IVP has a unique solution.

2) $(y^2+1)\,dx = y\sec^2 x\,dy$

$\int \frac{y}{y^2+1}dy = \int \cos^2 x\,dx$

$\frac{1}{2}\ln(y^2+1) = \int \frac{1+\cos 2x}{2}dx$

$= \left(x + \frac{\sin 2x}{2}\right)\frac{1}{2} + C$

$\boxed{\ln(y^2+1) = x + \frac{\sin 2x}{2} + C}$

3) $\frac{1}{x}\frac{dy}{dx} - 2y = \frac{2x+1}{x}$

$\frac{dy}{dx} - 2xy = 2x+1$

An integrating factor is

$e^{\int -2x\,dx} = e^{-x^2}$

So, $\frac{d}{dx}\left(y e^{-x^2}\right) = (2x+1)e^{-x^2}$

$y e^{-x^2} = \int (2x+1)e^{-x^2}dx$

$= \int 2x e^{-x^2} + \int e^{-x^2}dx$

$= -e^{-x^2} + \int_0^x e^{-t^2}dt + C$

$\boxed{y = -1 + e^{x^2}\int_0^x e^{-t^2}dt + Ce^{x^2}}$

$x > 0$.