MATH 202.6 (Term 131)

Quiz 2 (Chap. 2.4-2.5) Duration: 20mn

Name: **ID number:**

1.)(5pts)Solve the exact DE: $(\sin^2(x+y)+\frac{x^2y^3}{3})dx+(-\frac{1}{2}\cos 2(x+y)+\frac{x^3y^2}{3})dy=0.$

2.)(5pts)Solve by substitution the DE: $\frac{dy}{dx}=\frac{x-y+5}{x-y}$.

Solution

1.) $M=\sin^2(x+y)+x^2y^3/3$

$\Rightarrow M_y = 2\cos(x+y)\sin(x+y)+x^2y^2$

$= \sin 2(x+y) + x^2y^2$

$N = -\frac{1}{2}\cos 2(x+y) + x^3y^2/3$

$\Rightarrow N_x = \sin 2(x+y) + x^2y^2$

$M_y = N_x \Rightarrow$ DE exact.

$\frac{\partial f}{\partial x} = \sin^2(x+y)+\frac{x^2y^3}{3}$, $\frac{\partial f}{\partial y} = -\frac{1}{2}\cos 2(x+y) + \frac{x^3y^2}{3}$

$f(x,y) = \frac{-\sin 2(x+y)}{4} + \frac{x^3y^3}{9} + h(x)$

$\frac{\partial f}{\partial x} = -\frac{\cos 2(x+y)}{2} + \frac{x^2y^3}{3} + h'(x) =$

$\sin^2(x+y) + \frac{x^2y^3}{3}$

Noting that $\sin^2(x+y) = \frac{1-\cos 2(x+y)}{2}$,

we find $h'(x) = \frac{1}{2}$

$h(x) = \frac{x}{2} + C$

$\Rightarrow \boxed{\frac{-\sin 2(x+y)}{4} + \frac{x^2y^3}{3} + \frac{x}{2} = C}$

2.) $\frac{dy}{dx} = \frac{x-y+5}{x-y}$

$u = x-y \Rightarrow \frac{du}{dx} = 1 - \frac{dy}{dx}$

We substitute into the DE, and we find

$1 - \frac{du}{dx} = \frac{u+5}{u}$

$\frac{du}{dx} = 1 - \frac{u+5}{u} = -\frac{5}{u}$

$\int u\,du = -\int 5\,dx$

$u^2 = -5x + C$

$\boxed{(x-y)^2 = -5x + C}$