MATH 202.9 (Term 131)

Quiz 3 (Sects. 4.2, 4.3) Duration: 20mn

Name: ID number:

1.) a.)(5pts)Use reduction of order to find a second solution of the DE y_2 of $(1-2x-x^2)y''+2(1+x)y'-2y=0$, given that $y_1=x+1$ is a solution of the DE.

2.)(5pts)Solve the IVP $\begin{cases} y'''+y''+y'+y=0 \\ y(0)=y'(0)=0,\ y''(0)=2. \end{cases}$

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1) $y''+\frac{2(1+x)}{1-2x-x^2}y'-\frac{2}{1-2x-x^2}y=0$

$P(x)=\frac{2(1+x)}{1-2x-x^2}$

$y_2=y_1(x)\int\frac{e^{-\int P(x)dx}}{y_1^2(x)}$

$=(x+1)\int\frac{e^{-\int\frac{2(1+x)}{1-2x-x^2}dx}}{(x+1)^2}dx$

$=(x+1)\int\frac{e^{\ln|1-2x-x^2|}}{(x+1)^2}dx$

$=(x+1)\int\frac{1-2x-x^2}{(x+1)^2}dx$, $1-2x-x^2>0$

but, $1-2x-x^2=2-(1+x)^2$

Thus, $y_2(x)=(x+1)\int\left(\frac{2}{(x+1)^2}-1\right)dx$

$=(x+1)\left[-\frac{2}{x+1}-x\right]$

$=-2-x(x+1)$

2) The auxiliary equation is

$m^3+m^2+m+1=0$

$m^2(m+1)+(m+1)=0$

$(m+1)(m^2+1)=0$

$m=-1 \quad | \quad m=\pm i$

$y=c_1e^{-x}+c_2\cos x+c_3\sin x$

$y'=-c_1e^{-x}-c_2\sin x+c_3\cos x$

$y''=c_1e^{-x}-c_2\cos x-c_3\sin x$

$y(0)=0 \Rightarrow c_1+c_2=0$ (1)

$y'(0)=0 \Rightarrow -c_1+c_3=0$ (2)

$y''(0)=2 \Rightarrow c_1-c_2=2$ (3)

(1) $\Rightarrow c_2=-c_1$

(3) $\Rightarrow c_1+c_1=2 \Rightarrow c_1=1$

$c_2=-1$

(2) $\Rightarrow$ $c_3=c_1=1$

$\Rightarrow y=e^{-x}-\cos x+\sin x$