MATH 202.6 (Term 131)

Quiz 4 (Sects. 4.5, 4.6) Duration: 20mn

Name: ID number:

1.)(5pts)Solve the ODE $y'' - 4y = 4e^{5x} + 7\cos 2x$ by using annihilator approach.

2.)(5pts)Solve the ODE $y'' + 3y' + 2y = \frac{1}{e^x+1}$ by using variations of parameters.

1.) $m^2 - 4 = 0$, $m = \pm 2$

$\Rightarrow y_c = c_1 e^{-2x} + c_2 e^{2x}$

$(D-5)(D^2+4)\left[4e^{5x} + 7\cos 2x\right] = 0$

$(D^2-4)(D-5)(D^2+4)\,y = 0$

$m^2 = 4$, $m = 5$, $m = \pm 2i$

$\Rightarrow y = \underbrace{c_1 e^{-2x} + c_2 e^{2x}}_{y_c} + \underbrace{c_3 e^{5x} + c_4 \cos 2x + c_5 \sin 2x}_{y_p}$

The form of y_p is

$y_p = A e^{5x} + B\cos 2x + C \sin 2x$

$y_p' = 5A e^{5x} - 2B \sin 2x + 2C \cos 2x$

$y_p'' = 25A e^{5x} - 4B \cos 2x - 4C \sin 2x$

$y_p'' - 4y_p = 4e^{5x} + 7\cos 2x \Rightarrow$

$25Ae^{5x} - 4B\cos 2x - 4C\sin 2x$
$- 4Ae^{5x} - 4B\cos 2x - 4C\sin 2x =$
$4e^{5x} + 7\cos 2x$

$\Rightarrow 21A = 4 \longrightarrow A = 4/21$

$-8B = 7 \longrightarrow B = -7/8$

$-8C = 0 \longrightarrow C = 0$

$$\boxed{y = c_1 e^{-2x} + c_2 e^{2x} + \frac{4}{21} e^{5x} - \frac{7}{8}\cos 2x}$$

2.) $m^2 + 3m + 2 = 0$, $D = 9 - 8 = 1$

$m_1 = \frac{-3-1}{2} = -2$, $m_2 = \frac{-3+1}{2} = -1$

$\Rightarrow y_c = c_1 \underbrace{e^{-x}}_{y_1} + c_2 \underbrace{e^{-2x}}_{y_2}$

$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x}$

$u_1' = \frac{-y_2 f(x)}{W} = +\frac{e^{-2x}}{e^{-3x}}\left(\frac{1}{e^x+1}\right) = \frac{e^x}{e^x+1}$

$u_1 = \int \frac{e^x}{e^x+1}\,dx = \ln(e^x+1)$

$u_2' = \frac{y_1 f(x)}{W} = -\frac{e^{-x}}{e^{-3x}}\left(\frac{1}{e^x+1}\right) = -\frac{e^{2x}}{e^x+1}$

$u_2 = -\int \frac{e^{2x}}{e^x+1}\,dx$

$v = e^x$, $dv = e^x\,dx$

$u_2 = -\int \frac{v}{v+1}\,dv = -\int \left(1 - \frac{1}{v+1}\right)dv$

$= -\left[v - \ln(v+1)\right] = -\left[e^x - \ln(e^x+1)\right]$

$\Rightarrow y_p = e^{-x}\ln(e^x+1) - e^{-2x}\left[e^x - \ln(e^x+1)\right]$

$= -e^{-x} + (e^{-x} + e^{-2x})\ln(e^x+1)$

$$\Rightarrow \boxed{y = c_1 e^{-x} + c_2 e^{-2x} - e^{-x} + (e^{-x} + e^{-2x})\ln(e^x+1)}$$