**Question 1:** Mark the correct sentence as (TRUE) and the incorrect sentence as (FALSE).

(a) The set \( S = \{ X_i : X_i = \{(i, i^2), (i, (i-1)^2)\}, i = 1, 2\} \) is NOT a partition of the set \( A \times B \) where \( A = \{-1, 2\} \) and \( B = \{1, 4\} \).

(b) Let \( A \) and \( B \) be two sets such that \( A \subset B \). Then there is NO set in \( \mathcal{P}(B) \) that is disjoint from \( A \).

(c) \( [(P \land Q) \Rightarrow \lnot R] \equiv (\lnot P) \lor (\lnot Q) \lor (\lnot R) \) (You can do this without a truth table)

(d) Let \( x, y \in \mathbb{Z} \). Then the contrapositive of the implication: “If \( x^2 + 1 \) is even, then \( x \) is odd or \( y \) is even” is “If \( x \) is even or \( y \) is odd, then \( x^2 + 1 \) is odd”.

(e) If every month consists of 40 days, then every week consists of 7 days.

(f) Let \( S \) be some domain. To prove the statement \( \forall x \in S, P(x) \Rightarrow Q(x) \) by contradiction we prove that \( \exists x \in S, \lnot ([\lnot Q(x)) \Rightarrow (\lnot P(x))] \).
Question 2:

(a) If $\bigcap_{\alpha \in I} A_{\alpha} = \bigcup_{\alpha \in I} A_{\alpha}$ for some indexed collection of sets $\{A_{\alpha}\}_{\alpha \in I}$ with and indexed set $I$. What can you say about the relationship between the sets $A_{\alpha}$.

(b) Let $n \in \mathbb{N}$, and define the sets $A_n = \{0, \pm1, \pm2, \ldots, \pm n\}$. Then

(i) $\bigcap_{i \in \mathbb{N}} A_i =$

(ii) $\bigcup_{i \in \mathbb{N}} A_i =$
**Question 3:** Show whether the following statement is a tautology or not.

\[ (P \land R) \Rightarrow Q \] \iff \[ P \Rightarrow (R \Rightarrow Q) \]
Question 4:
(a) Each of the following describes an implication. Write the implication in the form “if, then.”

(i) Any point on the straight line with equation \(2y + x - 3 = 0\) whose \(x\) coordinate is an integer also has an integer for its \(y\) coordinate.

(ii) Let \(n \in \mathbb{Z}\). Whenever \(3n + 7\) is even, \(n\) is odd.

(iii) For an integer to be odd, it is sufficient that its square be odd.

(iv) A matrix \(A\) is invertible only if \(\det A \neq 0\).

(b) For the open sentences \(P(x) : |x + 1| < 2\) and \(Q(x) : x \in (-3, 1)\) over the domain \(\mathbb{R}\). State the biconditional \(P(x) \iff Q(x)\) in two ways: one using “if and only if” and the other using “necessary and sufficient”.
Question 5: Let $A$ and $B$ be two sets. Then

(a) Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

(b) Disprove that $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$. 
Question 6:

(a) For an integer \( x \), prove that \( 3 \mid x^2 \) if and only if \( 3 \mid x \).

(b) Prove that \( \sqrt{3} \) is an irrational number.
Question 7: Let $x$ be a positive real number. Prove that if $x - \frac{2}{x} > 1$, then $x > 2$ by

(a) a proof by contrapositive.

(b) a proof by contradiction
Question 8:
(a) Prove that there is no integer \( n \) such that \( n \equiv 5 \pmod{14} \) and \( n \equiv 3 \pmod{21} \).

(b) Evaluate the proposed proof of the following result.

**Result:**
If \( x \) is an irrational number and \( y \) is a rational number, then \( z = x - y \) is irrational.

**Proof:**
Assume, to the contrary, that \( z = x - y \) is rational. Then \( z = \frac{a}{b} \), where \( a,b \in \mathbb{Z} \) and \( b \neq 0 \). Since \( \sqrt{3} \) is irrational, we let \( x = \sqrt{3} \). Since \( y \) is rational, \( y = \frac{c}{d} \), for some integers \( c,d \) with \( d \neq 0 \). Therefore,

\[
\sqrt{3} = x = y + z = \frac{c}{d} + \frac{a}{b} = \frac{ad + bc}{bd}.
\]

Since \( ad + bc \) and \( bd \) are integers, where \( db \neq 0 \), it follows that \( \sqrt{3} \) is rational, producing a contradiction.