MATH 302: EXAM II, SEMESTER (131), NOVEMBER 25, 2013

Time: 08:00 to 10:00 pm

Name : .......................................... ........... ...........

ID : ..........................................

Section : ...........

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<th>Exercise</th>
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Exercise 1. Let \( f(x, y) = x^2 + y \).

(a) Find the directional derivative of \( f(x, y) \) in the direction of the vector \( \mathbf{v} = (1, 1) \).

(b) Find the equations of the normal line and the tangent plane to the surface \( z = f(x, y) = x^2 + y \) at the point \( (0, 1, 1) \).
Exercise 2. Let $C$ be the curve given by $y = x^2 + 1$, from $(0, 1)$ to $(2, 5)$. Evaluate the line integral with respect to arc length

$$I = \int_C x^2 \, ds$$
Exercise 3. Let $C$ be the positively oriented boundary of the region in the first quadrant consisting of all points between the two curves $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(a) Graph the path $C$.

(b) Using Green’s theorem, evaluate the work done by the force

$$ F(x, y) = \left( \frac{1}{3} y^3 - \sin(x)e^{x^2} \right) \mathbf{i} + (xy + xy^2 + y^3 \cos(y)) \mathbf{j} $$

in moving a particle along the path $C$. 
Exercise 4. Let $F$ be the vector field given by
\[ F(x, y) = (y e^{xy} + y^2 - 6xy + 6)i + (xe^{xy} + 2xy - 3x^2 - 2y)j. \]

(a) Show that $F$ is conservative.

(b) Find a potential of $F$.

(c) Evaluate the line integral $\int_C F \cdot dr$, where $C$ is the positively oriented quarter circle joining $(2, 0)$ and $(0, 2)$. 
Exercise 5. Let $S$ be the portion of the surface $z = x^2 - y^2$ in the first octant that is within the cylinder $x^2 + y^2 = 9$.

(a) Find the surface area of $S$. 

(b) Evaluate the surface integral
\[ \iint_S \frac{xy}{x^2 + y^2} \, dS \]
Exercise 6. Let $S$ be the surface given by

$$z = (1 - x)^2, \quad 0 \leq x \leq 1, \quad -1 \leq y \leq 1.$$  

Suppose that $S$ is upward oriented and let $C$ be the boundary of $S$. The path $C$ has a positive orientation with respect to the upward orientation of $S$.

Verify Stokes theorem

$$\oint C F \cdot dr = \iint_S (\text{Curl}(F) \cdot n) \, dS,$$

where $F(x, y, z) = (xy, yz, xz)$.