Important Instructions

1. Write your name and ID number on each sheet that you use.
2. At the end of the exam, place all your answer sheets in good order in a bundle. The instructor will staple these together.
3. Calculators are allowed, but not programmable calculators.
4. Mobiles must be switched off at all times during the exams; they must be placed in front of the student at all times.
5. Food is not allowed. Drinks are allowed.
6. Answer all questions. QUESTIONS ARE NOT EQUALLY WEIGHTED.

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ID:

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513 Exam 2, Questions

(The questions are not equally weighted)

1. If the Fourier transform of,

\[ f(x) = \begin{cases} 
0 & \text{for } |x| > 1 \\
-1 & \text{for } -1 \leq x < 0 \\
+1 & \text{for } 0 \leq x < 1 
\end{cases} \]

is \( F(k) = \frac{2}{ik\sqrt{2\pi}} (1 - \cos k) \), then evaluate the integral,

\[ \int_{-\infty}^{\infty} \left( \frac{1 - \cos k}{k} \right)^2 dk. \]

[6 points]

2. If \( F(w) = \frac{\pi^2}{4i} \left( e^{-2|w-2|} - e^{-2|w+2|} \right) \) is the Fourier transform of \( f(t) \), find the simplest expression in integral form for \( f(t) \) -- (but do not evaluate it).

[7 points]

3. If \( A = \begin{pmatrix} 1 & 0 & 2 \\
-1 & 1 & 3 \\
0 & 2 & 2 \end{pmatrix} \), calculate \( A^2 \). Then without any further direct matrix multiplication, find \( A^4 \).

[8 points]

4. Find \( e^A \), where \( A \) is the same matrix as in question 3 above.

[8 points]
5. Find the particular solution of the following ordinary differential equation using Fourier Transform methods,

\[ y'' + 2y' + 3y = te^{-2t}H(t) \]

[25 points]

6. (a) Using linear algebra methods only, solve the homogeneous system of differential equation,

\[
\begin{align*}
    x'_1 &= 3x_1 - 3x_3 \\
    x'_2 &= -x_1 + 2x_2 \\
    x'_3 &= 4x_1 - 3x_3
\end{align*}
\]

(b) Using linear algebra methods only, solve the non-homogenous system of differential equation,

\[
\begin{align*}
    x'_1 &= 3x_1 - 2x_2 + 1 + t \\
    x'_2 &= 4x_1 - x_2 + 1 + 2t
\end{align*}
\]

given that \( x_1(0) = -1/5 \) and \( x_2(0) = 2/5 \).  

[32 points]

7. Find the Fourier cosine series for,

\[
f(x) = \begin{cases} 
  0 & 0 \leq x \leq 1/2 \\
  x - 1/2, & 1/2 < x \leq 3/4 \\
  1 - x & 3/4 < x \leq 1 
\end{cases}
\]

Draw a sketch of the answer.

Without any further calculations, draw sketches of what you would expect, (a) the Fourier sine series of \( f(x) \) to look like and, (b) the Fourier series of \( f(x) \) to look like.

[32 points]
8. Find the general solution of the following ordinary differential equation using Fourier series methods,

\[ y'' + 4y = f(t), \]

\[ f(t) = t^2, \quad -\pi \leq t \leq \pi, \quad f(t + 2\pi) = f(t) \]

[32 points]

N Malik