King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 513 Mathematical Methods for Engineers
Exam 3 Term 131
Section 02 Instructor: Dr Nadeem Malik

December 31, 2013
Time: 6 – 9 pm (3 hours)

Important Instructions

1. Write your name and ID number on each sheet that you use.
2. At the end of the exam, place all your answer sheets in good order in a bundle. The instructor will staple these together.
3. Calculators are allowed, but not programmable calculators.
4. Mobiles must be switched off at all times during the exams; they must be placed in front of the student at all times.
5. Food is not allowed. Drinks are allowed.

NAME:
ID:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/30</td>
</tr>
<tr>
<td>2</td>
<td>/30</td>
</tr>
<tr>
<td>3</td>
<td>/30</td>
</tr>
<tr>
<td>4</td>
<td>/30</td>
</tr>
<tr>
<td>5</td>
<td>/30</td>
</tr>
<tr>
<td>TOTAL</td>
<td>/150</td>
</tr>
</tbody>
</table>
1. Use the separation of variables method to solve the equation,

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 4, \quad 0 < y < 8
\]

with

\[
u(x,0) = 2x, \quad u(x,8) = 8, \quad u(4,y) = 8
\]

\[
u(0,y) = \begin{cases} 
0 & 0 < y < 1 \\
4(y-1) & 1 < y < 3 \\
8 & 3 < y < 8
\end{cases}
\]

[30 points]

2. A vibrating string satisfies the equation,

\[
\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2} - u, \quad 0 < x < \pi, \quad t > 0
\]

The string is held fixed at, \( x = 0 \) and \( x = \pi \). We also have

\[
\frac{\partial u(x,0)}{\partial t} = 0, \quad \text{and}
\]

\[
u(x,0) = \begin{cases} 
x & 0 \leq x \leq \pi/2 \\
\pi - x & \pi/2 \leq x \leq \pi
\end{cases}
\]

Use the separation of variables method to find the general solution for \( u(x, t) \).

[30 points]
3. Use the separation of variables method to solve,

\[
\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 4, \quad t > 0
\]

with boundary conditions, \( u(0, t) = u(4, t) = 0 \) for \( t > 0 \), and initial conditions,

\[
u(x, 0) = x(1 - x^2) \quad \text{for} \quad 0 < x < 4, \quad \text{and}
\]

\[
\frac{\partial u(x, 0)}{\partial t} = 0 \quad \text{for} \quad 0 < x < 4.
\]

[30 points]

4. Apples are stored in ships for export. An apple is modeled as a sphere of radius 4 cm, and initially at a temperature of \( u = 0 \). At time \( t = 0 \), the skins of the apples are taken to be \( \theta = 16 \ DEG \). Use the separation of variables method to solve the equation for the temperature distribution \( u(r, t) \),

\[
\frac{\partial u}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) + G, \quad 0 < r < 4, \quad t > 0
\]

where \( G > 0 \). Assume that the temperature remains finite at \( r = 0 \), and that a steady state distribution is eventually established.

(Express your answer for \( u \) in in terms of Fourier coefficients, \( a_n \). Show the integral expressions for \( a_n \) but do not evaluate them.)

[30 points]

5. The electrostatic potential, \( u(r,x) \), inside a closed cylinder of length 4 cm and radius 2 cm is described by the Laplace equation,

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial x^2}, \quad 0 < r < 2, \quad 0 < x < 4
\]

The base and lateral surface have the potential 0 while the upper surface has potential 10. Find \( u(r, x) \).

[30 points]

N Malik