Math 533 Mid Term Exam

Nov 12, 2013-11-08

Q 1) (a) Find all solutions of the equation $z^4 + z^2 + 1 = 0$. Give your answer in polar form.

(b) Find all solutions of $tan^{-1}(z) = w$.

Q 2) Prove that any complex analytic function f is conformal at points where p where $f'(p) \neq 0$

Q 3)(a) Let $f(z) = \frac{z-a}{1-z\bar{a}}$. Show that f maps the unit disc onto itself if a is inside the unit disc.

(b) What is the image of $\frac{z-a}{\bar{z}-\bar{a}}$

Q4) (a) Show that the geometric series $1 + z + \ldots z^n + \cdots$ converges uniformly on any closed disc of radius less than 1

(b) Show that the series $\varsigma(z) = \sum_1^\infty n^{-z}$ converges uniformly on any strip $1 + \epsilon \leq Re(z)$, where $\epsilon > 0$.

Q5) Show that if γ is any curve in the plane , then $\left|\int_\gamma f(z)dz\right| \leq \int_\gamma |f(z)||dz|$