

Q1)

a) Find the image of the strip $\frac{-\pi}{2} < \text{Im}(z) < \frac{\pi}{2}$ under the map $z \rightarrow e^z$

b) Find the image of the half plane $\text{Re}(z) > 0$ under the map $z \rightarrow \frac{z-1}{z+1}$

c) Use a) and b) to find the image of the strip $\frac{-\pi}{2} < \text{Im}(z) < \frac{\pi}{2}$ under the map $z \rightarrow \tanh \frac{z}{2}$. (15 points)

Q2)

a) Find all roots of the equation $(z - 1)^{10} = z^{10}$ (10 points)

b) Find all roots of the equation $\cos z = 2$ (5 points)

Q3)

a) Prove Liouville's theorem: If an analytic function is defined on all complex numbers and it is bounded, then it must be constant (10 points)

b) Use a) to prove that if an analytic function is defined on all complex numbers and its k -th derivative is bounded, then it must be a polynomial of degree at most k (10 points)

Q4)

Let f be analytic on a closed disc $(0, R)$. Use Cauchy Integral Formula to derive the Taylor series for f at 0 (20 points)

Q5)a) Use Residue Calculus to find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (10 points)

b) Find the residues at the singularities of the function $\frac{1}{z^2} \tan\left(\frac{\pi}{2}z\right)$ (10 points)