Math 101 <u>Quiz 2</u> 11/02/2014

<u>Ex1</u>: Use ε, δ definition of the limit to show that

$$\lim_{x \to 9} \sqrt{x-5} = 2$$

<u>Solution</u>: Let $0 < \varepsilon < 1$. We need to find $\delta > 0$ such th

if $|x-9| < \delta$ then $\left|\sqrt{x-5} - 2\right| < \varepsilon$.

If $\left|\sqrt{x-5} - 2\right| < \varepsilon$, then: $2 - \varepsilon < \sqrt{x-5} < 2 + \varepsilon$.
Squaring the last inequality implies:

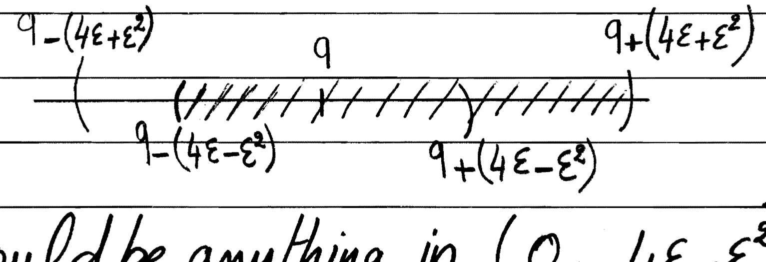
$$(2-\varepsilon)^2 < x - 5 < (2+\varepsilon)^2, \text{ and so:}$$

$$5 + (2-\varepsilon)^2 < x < 5 + (2+\varepsilon)^2$$

<u>or</u>

$$9 - (4\varepsilon - \varepsilon^2) < x < 9 + (4\varepsilon + \varepsilon^2).$$ In other words we have proved the following: for a fixed ε between 0 and

$$\left|\sqrt{x-5} - 2\right| < \varepsilon \quad \text{iff} \quad 9 - (4\varepsilon - \varepsilon^2) < x < 9 + (4\varepsilon + \varepsilon^2)$$



Hence δ could be anything in $(0, 4\varepsilon - \varepsilon^2]$. It is enough to choose $\delta = 4\varepsilon - \varepsilon^2$.

Ex2: $\lim\limits_{\theta \to 0^-} \dfrac{\sqrt{1-\cos 4\theta}}{3\theta}$ is of the form $\dfrac{0}{0}$

Now: $\sin^2(2\theta) = \dfrac{1-\cos(4\theta)}{2}$ and so:

$1-\cos 4\theta = 2\sin^2(2\theta)$. Thus:

$$\lim_{\theta \to 0^-} \frac{\sqrt{1-\cos 4\theta}}{3\theta} = \lim_{\theta \to 0^-} \frac{\sqrt{2\sin^2(2\theta)}}{3\theta} = \lim_{\theta \to 0^-} \frac{\sqrt{2}}{3} \frac{|\sin 2\theta|}{\theta}$$

Since $\theta \longrightarrow 0^-$, $\theta < 0$ and so $\sin(2\theta) < 0$. Thus:

$$\lim_{\theta \to 0^-} \frac{\sqrt{2}}{3} \frac{|\sin 2\theta|}{\theta} = \lim_{\theta \to 0^-} \frac{\sqrt{2}}{3} \frac{-\sin 2\theta}{\theta}$$

$$= \lim_{\theta \to 0^-} \left(-\frac{\sqrt{2}}{3} \cdot 2 \cdot \frac{\sin 2\theta}{2\theta} \right) = -\frac{2\sqrt{2}}{3}$$