Math 101 Quiz 5 06/03/2014

1) Let $f(x) = \frac{x^2+3e^x}{2e^x - x}$. Find $f'(0)$.

2) Find the normal line to the curve $f(x) = e^x(x-1)(x^2+1)$ at $x=1$

3) Find $\lim_{x\to 4} \frac{x^{3/2} - 8}{8-2x}$.

Solution

1) $f'(x) = \frac{(2x+3e^x)(2e^x - x) - (x^2+3e^x)(2e^x - 1)}{(2e^x - x)^2}$. Therefore:

$f'(0) = \frac{6-3}{4} = \frac{3}{4}$

2) $f'(x) = e^x(x-1)(x^2+1) + e^x(x^2+1) + e^x(x-1)2x$ Thus:

$f'(1) = 2e$. The slope of the normal line is $\frac{-1}{2e}$.
An equation of the normal line at $x=1$ is:

$$y = \frac{-1}{2e}(x-1) + \underbrace{f(1)}_{0} = \frac{1-x}{2e}$$

3) $$\lim_{x\to 4} \frac{x^{3/2} - 8}{8-2x} = \lim_{x\to 4} -\frac{1}{2}\frac{x^{3/2} - 8}{x-4} = -\frac{1}{2}\lim_{x\to 4}\frac{x^{\frac{3}{2}} - 4^{3/2}}{x-4}$$

$$= -\frac{1}{2}\cdot\frac{d}{dx}x^{3/2}\Big|_{x=4} = -\frac{1}{2}\cdot\frac{3}{2}x^{\frac{1}{2}}\Big|_{x=4}$$

$$= -\frac{1}{2}\frac{3}{2}\cdot 4^{1/2} = -\frac{3}{2}$$