Math 101 Quiz 7 01/04/2014

Find the derivative of y:

a) $y = \log_3\left[\left(\frac{2x+1}{3x-1}\right)^{\ln 3}\right]$

b) $y = 3^{\log_2 t}$

c) $y = (\sin x)^{\ln x}$

Solution

a) $y = \frac{1}{\ln 3} \cdot \ln\left[\left(\frac{2x+1}{3x-1}\right)^{\ln 3}\right] = \frac{\ln 3}{\ln 3} \ln\left(\frac{2x+1}{3x-1}\right) = \ln(2x+1) - \ln(3x-1).$

Hence $\frac{dy}{dx} = \frac{2}{2x+1} - \frac{3}{3x-1}$

b) $y = e^{\log_2 t \cdot \ln 3} = e^{\frac{\ln 3}{\ln 2} \cdot \ln t}$. Hence:

$\frac{dy}{dt} = \frac{1}{t} \cdot \frac{\ln 3}{\ln 2} \cdot e^{\frac{\ln 3}{\ln 2} \cdot \ln t} = \frac{3^{\log_2 t}}{t} \cdot \frac{\ln 3}{\ln 2}$

c) $y = (\sin x)^{\ln x} = e^{\ln x \cdot \ln(\sin x)}$. Therefore:

$\frac{dy}{dx} = \left[\frac{\ln(\sin x)}{x} + \ln x \frac{\cos x}{\sin x}\right] e^{\ln x \cdot \ln(\sin x)}$

$= \left[\frac{\ln(\sin x)}{x} + \ln x \cot x\right] (\sin x)^{\ln x}.$