MATH 102.6 (Term 132)

Quiz 1 (Sects. 5.3-5.5) Duration: 20mn

Name: ID number:

1.)(4pts)Use Riemann sum to write the integral $\int_0^1 (x^3 - 2x^2)dx$ as limit of sums. Find the value of this limit.

2.)(6pts)Evaluate the integrals $A = \int 60x^7\sqrt{x^4+1}dx$, $B = \int_{-1}^{0}(x+2)(x+1)^{99}dx$, $C = \int \frac{e^{2t}\tan(e^t)-1}{e^t}dt$

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1) $\Delta x = \frac{1}{n}$, $x_i^* = \frac{i}{n}$

$$\int_0^1 (x^3 - 2x^2)\,dx = \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n} (x_i^*)^3 - 2(x_i^*)^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^3}{n^3} - 2\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2 - \frac{2}{n^3}\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{1}{4} - \frac{4}{6} = -5/12$$

2) $A = \int 60x^7\sqrt{x^4+1}\,dx$

let $u = x^4+1$

$\Rightarrow du = 4x^3dx$

but $x^4 = u-1$

$\Rightarrow (u-1)\,du = 4x^7\,dx$

$$\Rightarrow A = 15\int (u-1)u^{1/2}\,du$$

$$= 15\left[\frac{2}{5}u^{5/2} - \frac{2u^{3/2}}{3}\right] + C$$

$$= 6(x^4+1)^{5/2} - 10(x^4+1)^{3/2} + C$$

$$B = \int_{-1}^{0}(x+2)(x+1)^{99}\,dx.$$

let $u = x+1 \Rightarrow du = dx$

$$\Rightarrow B = \int_0^1 (u+1)u^{99}\,du$$

$$= \left[\frac{u^{101}}{101} + \frac{u^{100}}{100}\right]_0^1$$

$$= \frac{1}{101} + \frac{1}{100} = \frac{201}{10100}$$

$$C = \int \frac{e^{2t}\tan e^t - 1}{e^t}\,dt$$

let $u = e^t \Rightarrow du = e^t dt$

and $dt = \frac{du}{u}$

$$\Rightarrow C = \int \frac{u^2\tan u - 1}{u^2}\,du$$

$$= \int\left(\tan u - \frac{1}{u^2}\right)du$$

$$= -\ln|\cos u| + \frac{1}{u} + C$$

$$= -\ln|\cos e^t| + \frac{1}{e^t} + C$$