MATH 102.5 (Term 132)

Quiz 6 (Sects. 10.4, 10.5 & 10.6) Duration: 20mn

Name: ID number:

1.)(2pts)Use comparison test to show that the series $\sum_{n=1}^{\infty} \frac{\sin^6(n^2-1)}{(n+2)^{4/3}}$ converges.

2.)(2pts)Use ratio test to study the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

3.)(4pts)Do the series $\sum_{n=1}^{\infty}\left(\frac{1+4\ln\sqrt{n}}{\sqrt{n}+3}\right)^n$ and $\sum_{n=1}^{\infty}\left(\frac{3+5n^2}{4n^2+n+1}\right)^n$ converge or diverge?

4.)(2pts)Find the smallest number of terms required to approximate the sum $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^4-2}$ so that $|\text{error}| < 0.0001$.

1-) $\frac{\sin^6(n^2+1)}{(n+2)^{4/3}} \leq \frac{1}{(n+2)^{4/3}}$

and the series $\sum_{n=0}^{\infty} \frac{1}{(n+2)^{4/3}}$ converges

$\Rightarrow \sum_{n=1}^{\infty} \frac{\sin^6(n^2+1)}{(n+2)^{4/3}}$ converges

2-) $\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \frac{n^n}{n!} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$

$\left(\frac{n}{n+1}\right)^n = e^{n\ln\left(\frac{n}{n+1}\right)}$

$\lim_{n\to\infty} n\ln\left(\frac{n}{n+1}\right) = \lim_{n\to\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}}$

$\left(\text{Hopital rule}\right) = \lim_{n\to\infty} \frac{\frac{1}{n(n+1)}}{-\frac{1}{n^2}}$

$= -1.$

$\Rightarrow \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = e^{-1} < 1$

$\Rightarrow \sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges

3.) $\sqrt[n]{a_n} = \frac{1+4\ln\sqrt{n}}{\sqrt{n}+3} = \frac{\ln\sqrt{n}}{\sqrt{n}} \cdot \frac{4+\frac{1}{\ln\sqrt{n}}}{1+\frac{3}{\sqrt{n}}}$

$\lim_{n\to\infty} \frac{\ln\sqrt{n}}{\sqrt{n}} = 0 \Rightarrow \lim_{n\to\infty} \sqrt[n]{a_n} = 0$

$\Rightarrow$ the series $\sum_{n=1}^{\infty}\left(\frac{1+4\ln\sqrt{n}}{\sqrt{n}+3}\right)^n$ converges

$\sqrt[n]{b_n} = \frac{3+5n^2}{4n^2+n+1}$

$\lim_{n\to\infty} \sqrt[n]{b_n} = 5/4 > 1$

$\Rightarrow$ the series $\sum_{n=1}^{\infty}\left(\frac{3+5n^2}{4n^2+n+1}\right)^n$ diverges

4.) We solve

$\frac{1}{(n+1)^4-2} < 0.0001 = 10^{-4}$

$\Rightarrow (n+1)^4 - 2 > 10^4$

$\Rightarrow (n+1)^4 > 10^4 + 2 = 10^4\left(1+\frac{2}{10^4}\right)$

$\Rightarrow n+1 > 10\sqrt[4]{1+2\cdot 10^{-4}}$

$\sqrt[4]{1+2\cdot 10^{-4}} = 1,0001$

$n+1 > 10,001$

$n \geq 10$ is the smallest integer number