Exercise | Points  
---|---
1 | 15  
2 | 12  
3 | 15  
4 | 8  
Total | 50
Exercise 1. Let $a, b, c, d$ be complex numbers. Evaluate the following determinants:

(i) $V(a, b) = \begin{vmatrix} 1 & a \\ 1 & b \end{vmatrix}$

(ii) $V(a, b, c) = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

(iii) $V(a, b, c, d) = \begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{vmatrix}$
Exercise 2. Let $A$ be a nonsingular $n \times n$-matrix with $n > 1$.

(a) Show that
\[ \det(\text{adj}(A)) = (\det(A))^{n-1}. \]

(b) Show that $\text{adj}(A)$ is nonsingular and
\[ (\text{adj}(A))^{-1} = \det(A^{-1})A = \text{adj}(A^{-1}). \]

(c) Show that a square matrix $M$ is singular if and only if $\text{adj}(M)$ is also singular.
Exercise 3. Let $A$ be a $k \times k$-matrix and $B$ be an $(n-k) \times (n-k)$-matrix.

Consider the matrices $E = \begin{pmatrix} I_k & O \\ O & B \end{pmatrix}$, $F = \begin{pmatrix} A & O \\ O & I_{n-k} \end{pmatrix}$ and $C = \begin{pmatrix} A & O \\ O & B \end{pmatrix}$.

(a) Show that $\det(E) = \det(B)$. 
(b) Show that $\det(F) = \det(A)$. 
(c) Show that \( \det(C) = \det(A) \det(B) \).

(d) Let \( M = \begin{pmatrix} O & B \\ A & O \end{pmatrix} \). Show that \( \det(M) = (-1)^k \det(A) \det(B) \).
Exercise 4. Given the matrix
\[ A = \begin{pmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{pmatrix}. \]

(a) Evaluate \( \det(A) \).

(b) Find the values of \( k \) for which the above matrix \( A \) is invertible.