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<th>Exercise</th>
<th>Points</th>
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<td>1</td>
<td>12</td>
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<td>2</td>
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Exercise 1 (12 pts). Is the set of all triples of real numbers \((x, y, z)\) equipped with the standard vector addition but with scalar multiplication defined by \(k(x, y, z) = (k^2x, k^2y, k^2z)\) a vector space (on \(\mathbb{R}\))?
Exercise 2 (12 pts). Determine which of the following subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$.

(1) All vectors of the form $(a, 0, 0)$.

(2) All vectors of the form $(a, 1, 1)$.

(3) All vectors of the form $(a, b, c)$, where $b = a + c$.

(4) All vectors of the form $(a, b, c)$, where $b = a + c + 1$. 
Exercise 3 (12 pts). Let 

\[ V = \{ (x, y, z) \in \mathbb{R}^3 : 3x - y + 5z = 0 \} . \]

Find a basis of \( V \) and evaluate the dimension of \( V \).
Exercise 4 (12 pts). Use the Wronskian to show that the functions
\[ f_1(x) = e^x, f_2(x) = xe^x \text{ and } f_3(x) = x^2 e^x \]
are linearly independent vectors in the space \( C^3(\mathbb{R}) \).
Exercise 5 (14 pts).

(1) Find the transition matrix from $B = (1, x, x^2)$ to the basis $B' = (1, 1 + x, (1 + x)^2)$.

(2) Given any $p(x) = a + bx + cx^2$ in $\mathbb{P}_2$, find the coordinates of $p(x)$ with respect to the basis $B' = (1, 1 + x, (1 + x)^2)$. 
Exercise 6 (12 pts). Let

\[ A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & 4 & -3 & 0 \\ 1 & 1 & 1 & 5 \end{pmatrix} \]

Find a basis for the row space \( \text{RS}(A) \) of \( A \), a basis for the column space \( \text{CS}(A) \) of \( A \) and a basis for the nullspace \( \text{NS}(A) \). Verify that \( \text{dim(NS}(A)) = n - \text{rank}(A) \).
Exercise 7 (14 pts). We denote by $\mathbf{P}_i$ the set of all polynomials of degree less than or equal to $i$. Let $T_1 : \mathbf{P}_1 \rightarrow \mathbf{P}_2$ be the linear transformation defined by $T_1(p(x)) = xp(x)$ and let $T_2 : \mathbf{P}_2 \rightarrow \mathbf{P}_2$ be the linear operator defined by $T_2(p(x)) = p(2x + 1)$. Let $\mathcal{B} = \{1, x\}$ and $\mathcal{B}' = \{1, x, x^2\}$ be the standard bases for $\mathbf{P}_1$ and $\mathbf{P}_2$, respectively.

1. Find the matrix of $T_1$ with respect to the bases $\mathcal{B}$ and $\mathcal{B}'$.

2. Find the matrix of $T_2$ with respect to the basis $\mathcal{B}'$.

3. Find the matrix of $T_2 \circ T_1$ with respect to the bases $\mathcal{B}$ and $\mathcal{B}'$. 
Exercise 8 (12 pts). Let $B = (u_1, u_2, u_3)$ be a basis for a vector space $V$, and let $T : V \to V$ be the linear transformation with matrix relatively to $B$:

$$M = \begin{pmatrix} -3 & 4 & 7 \\ 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

(1) Set $v_1 = u_1$, $v_2 = u_1 + u_2$ and $v_3 = u_1 + u_2 + u_3$. Show that $B' = (v_1, v_2, v_3)$ is a basis of $V$.

(2) Find the matrix of $T$ with respect to $B'$. 
