Use Laplace Transform to solve

\[ u_{xx} = u_t, \quad x > 0, \quad t > 0, \quad u(0,t) = 0, \quad u(x,0) = e^{-x}, \quad u_t(x,0) = 0, \quad \lim_{x \to \infty} u_t(x,t) = 0 \]

[Note: \( \mathcal{L}(f'(x)) = s^2 F(s) - s f(0) - f'(0) \), \( \mathcal{L}(u(x-a)) = e^{-as} F(s) \)]