Prove that if \( A \) is a measureable function,

\[
\{ x \in A \} = \bigcup_{\alpha} \{ g(\alpha) \}
\]

for \( \alpha \in \mathcal{A} \). Let \( \mathcal{A} \) be a measure space, and suppose that \( \mathcal{A} \) is an element of \( \mathcal{B} \).

Problem 1:

Let \( f \) be a measurable function. Prove that \( \int f \) is measurable.

Problem 2:

Let \( X \) be a measure space and \( f : X \to \mathbb{R} \) a measurable function. Prove that \( \{ x \in X : f(x) < c \} \) is a measurable set.

Problem 3:

Let \( \mathcal{A} \) be a measurable function and \( f^a \) a sequence of measurable functions. Show that the set

\[
\{ A \} = \bigcup_{\alpha} \{ g(\alpha) \}
\]

Find conditions on \( \mathcal{B} \) ensuring that \( \mathcal{A} \mathcal{C} \mathcal{B} \).

Problem 4:

Let \( \mathcal{A} \) be a set and \( \mathcal{B} \) a measurable set. We define the class \( \mathcal{C} = \{ B \subseteq \mathcal{A} \} \).

Problem 5:

Let \( \mathcal{A} \) be a measure in \( \mathcal{B} \). Show that \( \liminf_{n \to \infty} f_n = f \).

Problem 6:

Let \( \mathcal{A} \) be a measure in \( \mathcal{B} \). Show that \( \mathcal{C} = \{ -\infty, 0, \infty \} \) and \( \mathcal{D} = \{ \mathcal{A} \} \).

Problem 7:

We define the characteristic function of \( \mathcal{A} \mathcal{C} \mathcal{B} \) in two different manners.

Problem 8:

Show that if \( g \) is a measurable function in \( \mathcal{B} \) then \( f = f \).

Problem 9:

Show that if \( \mathcal{A} \mathcal{C} \mathcal{B} \) is a measurable function, \( f \) is not necessarily equal to \( f \).

Problem 10:

Let \( \mathcal{A} \mathcal{C} \mathcal{B} \) be a measurable function, and \( f \) be not necessarily equal to \( f \).