I. Let \( F(u) = \int_{-\infty}^{\infty} \text{e}^{-ut} \, dt \), \( u > 0 \). Find the value of \( F(u) \) (in terms of \( u \)). Justify all your steps.

II. Compute the integrals:
   (a) \( \lim_{n \to \infty} \int_{-\infty}^{\infty} \frac{\sin x}{x(1+x^2)} \, dx \)
   (b) \( \lim_{n \to \infty} \int_{0}^{\infty} e^{-x} \text{arctan} \frac{x}{n} \, dx \)

III. (a) Prove that \( \forall \epsilon > 0 \), \( 0 \leq 1 - e^{-\epsilon^2} \leq \epsilon \)
    (b) Deduce that \( \forall y > 0 \), \( x \mapsto \frac{1 - e^{-xy}}{x^2} \) is integrable on \([0, \infty)\)
    (c) For \( y > 0 \), let \( F(y) = \int_{0}^{\infty} \frac{1 - e^{-xy}}{x^2} \, dx \). Prove that \( F \) is differentiable on \((0, \infty)\). Compute \( F'(y) \). We recall that \( \int_{0}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}/2 \).
    (d) Deduce \( F(y) \) modulo a constant.
    (e) Compute this constant by looking at \( \lim_{n \to \infty} F(\frac{1}{n}) \).

IV. Let \((E, \Sigma, \mu)\) be a measure space, \( f : E \to [0, \infty) \) a positive measurable function such that \( 0 < \int_{E} f \, d\mu < \infty \).
    Find, in terms of \( x \in \mathbb{R}^+ \), \( \lim_{n \to \infty} \int_{E} \ln \left( 1 + \left( \frac{f(x)}{n} \right)^x \right) \, d\mu(x) \).
    (Hint: \( 1 + t^x \leq (1 + t)^x \), \( t > 0 \), \( x > 1 \)).

V. Let \( f(x) = \int_{0}^{\infty} e^{-t^2} \cos(tx) \, dt \), \( x \in \mathbb{R} \).
   (1) Prove that \( f \in C'(\mathbb{R}) \).
   (2) Prove that \( f \) satisfies \( y' = -\frac{x}{2} y \).
   (3) Deduce an explicit expression of \( f \).