1. Let \((E, \Sigma, \mu)\) be a measure space. We define
\[ \overline{\Sigma} = \{ A \in \Sigma : \forall N \in \mathbb{N}, \exists A_N \in \Sigma, A = A_N, \forall n \in \mathbb{N}, \mu(A_N) = 0 \} \]
and
\[ \overline{\mu} : \overline{\Sigma} \to [0, \infty], \forall A \in \overline{\Sigma}, \forall N \in \mathbb{N}, \overline{\mu}(A \cup N) = \overline{\mu}(A) \]
then,
(a) \(\overline{\Sigma}\) is a \(\sigma\)-algebra of \(\Sigma\).
(b) \(\overline{\mu}\) is a complete positive measure extending \(\mu\) from \(\Sigma\) to \(\overline{\Sigma}\).
(c) The space \((E, \overline{\Sigma}, \overline{\mu})\) is the smallest complete measure space “containing” \((E, \Sigma, \mu)\).

2. Let \((E, \Sigma, \mu)\) be a measure space, and \((E, \overline{\Sigma}, \overline{\mu})\) its completion. If \(\mu^*\) is the outer measure generated by \(\mu\) and \(B_{\mu^*}\) the \(\sigma\)-algebra of \(\mu^*\) measurable sets, then
\[ \overline{\Sigma} \subseteq B_{\mu^*} \text{ and } \overline{\mu^*} |_{\overline{\Sigma}} = \overline{\mu} \]

3. Let \((E, \Sigma, \mu)\) be a finite measure space and \(\mu^*\) is the outer measure extending \(\mu\) to \(\mathcal{P}(E)\). Prove that
\[ A \in B_{\mu^*} \iff \mu^*(E) = \mu^*(A) + \mu^*(A^c) \]

4. Prove that
(i) If \(A \subseteq \mathbb{R}\) is such that \(\mu^*(A) = 0\) then \(A \subseteq \mathbb{Q}\).
(ii) \(\mu^*\) is complete.
(iii) \(B(\mathbb{R}) \subseteq L \subseteq \mathcal{P}(\mathbb{R})\).
(iv) \(\mu(A + x) = \mu(A) + x \in \mathbb{R}, A \subseteq \mathbb{R}\).
(v) \(\mu(\lambda A) = |\lambda| \mu(A) + x \in \mathbb{R}, A \subseteq \mathbb{R}\).