Prove that \( f \) is Borel measurable.

Let \( f \) be a Borel measurable function.

(A) Show that the set \( B = \{ x \in A \mid x = \text{int } x \} \) is also a Borel set.

Let \( A \) be a Borel subset of \( \mathbb{R} \) and \( m(A) = 0 \).

Define \( F = \bigcup_{n=1}^{\infty} F_n \). Show that \( m(F) = 0 \). Let \( \{ F_n \} \) be a sequence of measurable functions.

Prove that \( \chi \) is a discountion function such that \( \chi = \chi^+ - \chi^- \) where \( \chi^+ = \sup_{x \in E} \chi \) and \( \chi^- = \inf_{x \in E} \chi \).

Verify the following definitions:

- \( \chi(x) = \chi^+ - \chi^- \) for all \( x \in E \).
- \( \chi(x) = \chi^+ \) if \( x \in E^+ \).
- \( \chi(x) = \chi^- \) if \( x \in E^- \).

Let \( x^* : \mathbb{R} \to [-\infty, +\infty] \) be defined by

\[ x^*(x) = \sup_{t \in \mathbb{R}} (c, t) \epsilon E. \]

\[
\text{HW #3}
\]