Problem 1 (25 pts)

Given the following pair of linear programs:

\[
\text{max}_{x,y} \quad z = 2x + 3y \quad \text{min}_{\alpha,\beta} \quad \gamma = 5\alpha + 4\beta \\
\text{s.t.} \quad x + 2y \leq 5, \quad \text{s.t.} \quad -\alpha - 2\beta \leq -2 \\
\quad 2x + y \leq 4, \quad \quad 2\alpha + \beta \geq 3 \\
\quad x, y \geq 0 \quad \quad \alpha, \beta \geq 0
\]

(a) What is the relation between these two linear programs. (5pts)

(b) Write the linear complementary conditions corresponding to these two linear programs. (10pts)

(c) Solve both programs graphically and give their optimal solutions. (10pts)
Problem 2 (40 pts)

Consider the following linear program:

\[
\begin{align*}
\text{max} & \quad 3x_1 + 2x_2 + x_3 \\
\text{s.t.} & \quad x_1 + x_2 + 2x_3 \leq 3, \\
& \quad x_1 - x_2 + x_3 \geq 2, \\
& \quad 2x_1 + x_2 + x_3 \leq 4, \\
& \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

(a) Solve the linear program using the Primal Simplex algorithm.(20pts)

(b) Solve the linear program using the Dual Simplex algorithm.(20pts)
Problem 3 (35 Points)

Given the following linear program:

\[
\begin{align*}
\max_{x_1, x_2, x_3} & \quad 3x_1 + 5x_2 + 2x_3 \\
\text{s.t.} & \quad 2x_1 + 4x_2 + x_3 \leq 7, \\
& \quad 3x_1 + 2x_2 + x_3 \leq 4, \\
& \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

(a) Write the standard form corresponding to the linear program. (5pts)

(b) Solve the linear program using the Revised Simplex method. (30pts)