1. The function $f(x) = \begin{cases} x^2 + b\,x & \text{if} \quad x \le 1 \\ a\,x + b & \text{if} \quad x > 1 \end{cases}$

   is differentiable everywhere. Then $a + b =$

   a) 0
   b) 2
   c) -2
   d) 1
   e) -1

2. If there are two tangent lines from the point $P(0,0)$ that touch the graph of $f(x) = x^2 + x + 1$ at $x = a$ and $x = b$, then $a\,b =$

   a) -1
   b) 1
   c) 0
   d) 2
   e) -2

3. Using the graph of $f(x) = e^x$, the maximum value of δ such that $|f(x) - 1| < 0.1$ whenever $|x| < \delta$ is equal to

   a) $\ln 11 - \ln 10$
   b) $\ln 11 - \ln 9$
   c) $\ln 10 - \ln 9$
   d) $\ln 11$
   e) $\ln 10$

4. If $f(x) = (x^2 - 3\,x)^{40}$, then $f^{(80)}(2) =$

   a) $80\,!$
   b) 80
   c) 0
   d) $3(79\,!)$
   e) $2(40\,!)$

5. If $h(x) = \dfrac{\csc x}{g(x)+1}$ with $h'\left(\frac{\pi}{2}\right) = 2$ and $g\left(\frac{\pi}{2}\right) = 1$, then $g'\left(\frac{\pi}{2}\right)$ is

a) -8
b) 8
c) 4
d) -4
e) 0

6. If $y = \tan^2 x$, then $y'' =$

a) $6y^2 + 8y + 2$
b) $6y^2 + 8y$
c) $6y^2 + 4y + 2$
d) $6y^2 + 4y$
e) $6y^2 - 8y + 1$

7. The slant asymptote of $f(x) = 2e^{-x} - 2x + 3$ is

   a) $y = -2x + 3$
   b) $y = 2x - 3$
   c) $y = -2x$
   d) $y = x$
   e) $y = -2x + 5$

8. A particle is moving along the curve $y = \tan^{-1} x$. As the particle passes through the point $(1, \frac{\pi}{4})$, its x-coordinate increases at a rate of 2 cm/s. The rate of change of the distance from the particle to the origin at that instant is

   a) $\dfrac{8+\pi}{\sqrt{16+\pi^2}}$ cm/s
   b) $\dfrac{8+\pi}{2\sqrt{16+\pi^2}}$ cm/s
   c) $\dfrac{4+\pi}{4\sqrt{16+\pi^2}}$ cm/s
   d) $\dfrac{2+\pi}{\sqrt{16+\pi^2}}$ cm/s
   e) $\dfrac{4+\pi}{\sqrt{16+\pi^2}}$ cm/s

9. Using linear approximation, the number $\sqrt[3]{9}$ is estimated as

a) $\dfrac{25}{12}$

b) $\dfrac{24}{12}$

c) $\dfrac{23}{12}$

d) $\dfrac{22}{12}$

e) $\dfrac{21}{12}$

10. If $f'(x)$ is a continuous function and $f'(2) = 5$, then $\lim\limits_{x \to 0} \dfrac{f(2+3x) - f(2-4x)}{x}$

a) 35

b) 30

c) 20

d) 25

e) 40

11. The position function of a particle moving along a straight line is $s(t) = 2t - t^2$ for t in $[0, 5]$, where t is measured in seconds and s in meters. The particle is speeding up when

a) $1 < t < 5$

b) $0 < t < 1$

c) $0 < t < 2$

d) $0 < t < 3$

e) $0 < t < 5$

12. The radius of a sphere is measured to be 3 cm with a maximum error in measurement of 0.1 cm. Using differentials, the maximum error in calculating volume of the sphere is

a) $\dfrac{36\pi}{10}$ cm^3

b) 36π cm^3

c) $\dfrac{9\pi}{10}$ cm^3

d) 9π cm^3

e) $\dfrac{\pi}{10}$ cm^3

13. If $xy^2 - 3x^2y = x - 3$, then y' and y'' at the point $(1, 1)$ are given as

a) $y' = -6,\ y'' = 114$
b) $y' = 6,\ y'' = -114$
c) $y' = -6,\ y'' = -112$
d) $y' = 6,\ y'' = 114$
e) $y' = -6,\ y'' = 112$

14. The slope of the line tangent to the curve $y = \sqrt[3]{\dfrac{(x+1)^5(x+2)^4}{x^2+11}}$ at the point $(1, 6)$ is

a) $\dfrac{22}{3}$
b) $\dfrac{44}{3}$
c) $\dfrac{11}{9}$
d) $\dfrac{11}{6}$
e) $\dfrac{25}{3}$

15. The graph of the function $y = x^2 \ln x$ has

a) absolute minimum $-\dfrac{1}{2e}$ at $x = \dfrac{1}{\sqrt{e}}$

b) absolute maximum $-\dfrac{1}{2e}$ at $x = \dfrac{1}{\sqrt{e}}$

c) two critical numbers $x = 0$ and $x = \dfrac{1}{\sqrt{e}}$

d) absolute maximum at $x = 0$ and absolute minimum at $x = \dfrac{1}{\sqrt{e}}$

e) no absolute extrema

16. The graph of the function $f(x) = x - 3 + \sin^2\left(\dfrac{x}{3}\right)$ crosses the x-axis at

a) one point

b) no points

c) two points

d) three points

e) infinite number of points

17. Which of the following is **FALSE** about the graph of the function $f(x) = \dfrac{x^2}{x^3 - 1}$?

a) The graph is decreasing on $\left(-\infty, -\sqrt[3]{2}\right) \cup (0, \infty)$

b) The graph is decreasing on $\left(-\infty, -\sqrt[3]{2}\right) \cup (0, 1) \cup (1, \infty)$

c) The graph has two critical numbers

d) The graph has one horizontal asymptote and one vertical asymbtote

e) The graph crosses or touches the x-axis at one point only

18. Which of the following is **TRUE** about the graph of the function $f(x) = x^{\frac{5}{3}} - x^{\frac{2}{3}}$?

a) The graph has one inflection point

b) The graph is concave up on $\left(-\infty, -\frac{1}{5}\right)$

c) The graph is concave down on $(-\infty, 0)$

d) The graph is concave down on $(0, \infty)$

e) The graph is concave down on $\left(-\frac{1}{5}, \infty\right)$

19. The value of the limit $\lim_{x \to 0} (1+2x)^{\cot x}$ equals

a) e^2

b) $\dfrac{1}{e}$

c) ∞

d) 1

e) Does not exist

20. A store can sell 80 bicycles a month at a price of \$100 each. For each \$10 increase in the price, 5 fewer bicycles will be sold each month. Certain price will result in the maximum monthly revenue from bicycles sales. The maximum monthly revenue is

a) \$8450

b) \$8500

c) \$8550

d) \$8400

e) \$8000

21. Use Newton's method to find the positive fourth root of 2 by solving the equation $x^4 = 2$. If you start with $x_0 = 1$ to find x_1 and x_2, then $x_1 - x_2 =$

a) $\frac{113}{2000}$

b) $-\frac{113}{2000}$

c) $\frac{2387}{2000}$

d) $\frac{5}{16}$

e) $-\frac{5}{16}$

22. $\int \left(4^{(x+1)} - \frac{3}{\sqrt{1-x^2}}\right) dx =$

a) $\frac{4^{(x+1)}}{\ln 4} + 3\cos^{-1} x + C$

b) $\frac{4^{(x+1)}}{\ln 4} + 3\sin^{-1} x + C$

c) $\frac{4^{(x+1)}}{\ln 4} - 3\cos^{-1} x + C$

d) $\frac{4^{x}}{\ln 4} + 3\sin^{-1} x + C$

e) $\frac{4^{x}}{\ln 4} + 3\cos^{-1} x + C$

23. Consider the area below the graph of $f(x) = x - x^2$ and above the x-axis. If we use the left endpoint rule and 5 subintervals to approximate this area, we get

a) $\frac{4}{25}$

b) $\frac{1}{5}$

c) $\frac{1}{4}$

d) $\frac{6}{25}$

e) $\frac{4}{5}$

24. The Riemann sum formula for the function $f(x) = 1 + x^2$ obtained by dividing the interval $[0, 3]$ into n equal subintervals and using the right endpoint rule is

a) $3 + \frac{9(n+1)(2n+1)}{2n^2}$

b) $\frac{9(n+1)(2n+1)}{2n^2}$

c) $3 + \frac{3(n+1)(2n+1)}{2n^2}$

d) $\frac{3(n+1)(2n+1)}{2n^2}$

e) $3 + \frac{9(n+1)^2}{2n^2}$

25. If $f(3) = 2$ and $f'(x) \geq 3$ for $2 \leq x \leq 3$, then the largest possible value of $f(2)$ is

a) -1

b) 1

c) $\frac{1}{3}$

d) $\frac{1}{6}$

e) 0

26. If c is a number satisfying the conclusion of the Mean Value Theorem when applied to $f(x) = \ln x$ on $[1, e]$, then $c =$

a) $e - 1$

b) $1 - e$

c) e

d) $2e$

e) $e - 2$

27. The sum of the absolute maximum value and the absolute minimum value of the function $f(x) = \sin x + \cos x + 1$ on the internal $[0, \frac{\pi}{2}]$ is

a) $3 + \sqrt{2}$
b) $1 + \sqrt{2}$
c) $2 + \sqrt{2}$
d) $1 + \dfrac{1}{\sqrt{2}}$
e) $\dfrac{1}{\sqrt{2}}$

28. If (a, b) is a point on the hyperbola $x^2 - y^2 = 4$ which is closest to the point $(0, 1)$, then $b =$

a) $\dfrac{1}{2}$
b) $\dfrac{1}{4}$
c) 11
d) 9
e) $\dfrac{1}{11}$