1. The function $f(x) = \begin{cases} 
  x^2 + bx & \text{if } x \leq 1 \\
  ax + b & \text{if } x > 1 
\end{cases}$

is differentiable everywhere. Then $a + b =$

a) 0  
b) 2  
c) $-2$  
d) 1  
e) $-1$

2. If there are two tangent lines from the point $P(0,0)$ that touch the graph of $f(x) = x^2 + x + 1$ at $x = a$ and $x = b$, then $a b =$

a) $-1$  
b) 1  
c) 0  
d) 2  
e) $-2$
3. Using the graph of $f(x) = e^x$, the maximum value of $\delta$ such that $|f(x) - 1| < 0.1$ whenever $|x| < \delta$ is equal to

a) $\ln 11 - \ln 10$

b) $\ln 11 - \ln 9$

c) $\ln 10 - \ln 9$

d) $\ln 11$

e) $\ln 10$

4. If $f(x) = (x^2 - 3x)^{40}$, then $f^{(80)}(2) =$

a) $80!$

b) 80

c) 0

d) $3(79!)$

e) $2(40!)$
5. If \( h(x) = \frac{\csc x}{g(x) + 1} \) with \( h'(\frac{\pi}{2}) = 2 \) and \( g'(\frac{\pi}{2}) = 1 \), then \( g'(\frac{\pi}{2}) \) is

a) -8
b) 8
c) 4
d) -4
e) 0

6. If \( y = \tan^2 x \), then \( y'' = \)

a) \( 6y^2 + 8y + 2 \)
b) \( 6y^2 + 8y \)
c) \( 6y^2 + 4y + 2 \)
d) \( 6y^2 + 4y \)
e) \( 6y^2 - 8y + 1 \)
7. The slant asymptote of \( f(x) = 2e^{-x} - 2x + 3 \) is

a) \( y = -2x + 3 \)
b) \( y = 2x - 3 \)
c) \( y = -2x \)
d) \( y = x \)
e) \( y = -2x + 5 \)

8. A particle is moving along the curve \( y = \tan^{-1} x \). As the particle passes through the point \( (1, \frac{\pi}{4}) \), its \( x \)-coordinate increases at a rate of 2 cm/s. The rate of change of the distance from the particle to the origin at that instant is

a) \( \frac{8 + \pi}{\sqrt{16 + \pi^2}} \) cm/s
b) \( \frac{8 + \pi}{2\sqrt{16 + \pi^2}} \) cm/s
c) \( \frac{4 + \pi}{4\sqrt{16 + \pi^2}} \) cm/s
d) \( \frac{2 + \pi}{\sqrt{16 + \pi^2}} \) cm/s
e) \( \frac{4 + \pi}{\sqrt{16 + \pi^2}} \) cm/s
9. Using linear approximation, the number $\sqrt[3]{9}$ is estimated as

a) $\frac{25}{12}$  
b) $\frac{24}{12}$  
c) $\frac{23}{12}$  
d) $\frac{22}{12}$  
e) $\frac{21}{12}$

10. If $f'(x)$ is a continuous function and $f'(2) = 5$, then $\lim_{x \to 0} \frac{f(2 + 3x) - f(2 - 4x)}{x}$

a) 35  
b) 30  
c) 20  
d) 25  
e) 40
11. The position function of a particle moving along a straight line is 
   
   \[ s(t) = 2t - t^2 \]
   
   for \( t \) in \([0, 5]\), where \( t \) is measured in seconds and \( s \) in meters. 
   The particle is speeding up when

   a) \( 1 < t < 5 \)
   b) \( 0 < t < 1 \)
   c) \( 0 < t < 2 \)
   d) \( 0 < t < 3 \)
   e) \( 0 < t < 5 \)

12. The radius of a sphere is measured to be 3 cm with a maximum error in measurement of 0.1 cm. Using differentials, the maximum error in calculating volume of the sphere is

   a) \( \frac{36\pi}{10} \) cm\(^3\)
   b) \( 36\pi \) cm\(^3\)
   c) \( \frac{9\pi}{10} \) cm\(^3\)
   d) \( 9\pi \) cm\(^3\)
   e) \( \frac{\pi}{10} \) cm\(^3\)
13. If $xy^2 - 3x^2y = x - 3$, then $y'$ and $y''$ at the point (1, 1) are given as

a) $y' = -6$, $y'' = 114$

b) $y' = 6$, $y'' = -114$

c) $y' = -6$, $y'' = -112$

d) $y' = 6$, $y'' = 114$

e) $y' = -6$, $y'' = 112$

14. The slope of the line tangent to the curve $y = \sqrt[3]{\frac{(x+1)^5(x+2)^4}{x^2+11}}$ at the point (1, 6) is

a) $\frac{22}{3}$

b) $\frac{44}{3}$

c) $\frac{11}{9}$

d) $\frac{11}{6}$

e) $\frac{25}{3}$
15. The graph of the function \( y = x^2 \ln x \) has

a) absolute minimum \(-\frac{1}{2e}\) at \( x = \frac{1}{\sqrt{e}} \)

b) absolute maximum \(-\frac{1}{2e}\) at \( x = \frac{1}{\sqrt{e}} \)

c) two critical numbers \( x = 0 \) and \( x = \frac{1}{\sqrt{e}} \)

d) absolute maximum at \( x = 0 \) and absolute minimum at \( x = \frac{1}{\sqrt{e}} \)

e) no absolute extrema

16. The graph of the function \( f(x) = x - 3 + \sin^2 \left( \frac{x}{3} \right) \) crosses the \( x \)-axis at

a) one point
b) no points
c) two points
d) three points
e) infinite number of points
17. Which of the following is **FALSE** about the graph of the function \( f(x) = \frac{x^2}{x^3 - 1} \)?

a) The graph is decreasing on \((−∞, −\sqrt{2}) \cup (0, ∞)\)
b) The graph is decreasing on \((−∞, −\sqrt{2}) \cup (0, 1) \cup (1, ∞)\)
c) The graph has two critical numbers
d) The graph has one horizontal asymptote and one vertical asymptote
e) The graph crosses or touches the x-axis at one point only

18. Which of the following is **TRUE** about the graph of the function \( f(x) = x^{\frac{5}{3}} - x^{\frac{2}{3}} \)?

a) The graph has one inflection point
b) The graph is concave up on \((−∞, −\frac{1}{5})\)
c) The graph is concave down on \((−∞, 0)\)
d) The graph is concave down on \((0, ∞)\)
e) The graph is concave down on \((−\frac{1}{5}, ∞)\)
19. The value of the limit $\lim_{x \to 0} (1 + 2x)\cot x$ equals

a) $e^2$

b) $\frac{1}{e}$

c) $\infty$

d) 1

e) Does not exist

20. A store can sell 80 bicycles a month at a price of $100 each. For each $10 increase in the price, 5 fewer bicycles will be sold each month. Certain price will result in the maximum monthly revenue from bicycles sales. The maximum monthly revenue is

a) $8450$

b) $8500$

c) $8550$

d) $8400$

e) $8000$
21. Use Newton’s method to find the positive fourth root of 2 by solving the equation \( x^4 = 2 \). If you start with \( x_0 = 1 \) to find \( x_1 \) and \( x_2 \), then \( x_1 - x_2 = \)

a) \( \frac{113}{2000} \)

b) \( -\frac{113}{2000} \)

c) \( \frac{2387}{2000} \)

d) \( \frac{5}{16} \)

e) \( -\frac{5}{16} \)

22. \( \int \left( 4^{(x+1)} - \frac{3}{\sqrt{1-x^2}} \right) dx = \)

a) \( \frac{4^{(x+1)}}{\ln 4} + 3 \cos^{-1} x + C \)

b) \( \frac{4^{(x+1)}}{\ln 4} + 3 \sin^{-1} x + C \)

c) \( \frac{4^{(x+1)}}{\ln 4} - 3 \cos^{-1} x + C \)

d) \( \frac{4^x}{\ln 4} + 3 \sin^{-1} x + C \)

e) \( \frac{4^x}{\ln 4} + 3 \cos^{-1} x + C \)
23. Consider the area below the graph of \( f(x) = x - x^2 \) and above the \( x \)-axis. If we use the left endpoint rule and 5 subintervals to approximate this area, we get

a) \( \frac{4}{25} \)

b) \( \frac{1}{5} \)

c) \( \frac{1}{4} \)

d) \( \frac{6}{25} \)

e) \( \frac{4}{5} \)

24. The Riemann sum formula for the function \( f(x) = 1 + x^2 \) obtained by dividing the interval \([0, 3]\) into \( n \) equal subintervals and using the right endpoint rule is

a) \( 3 + \frac{9(n + 1)(2n + 1)}{2n^2} \)

b) \( \frac{9(n + 1)(2n + 1)}{2n^2} \)

c) \( 3 + \frac{3(n + 1)(2n + 1)}{2n^2} \)

d) \( \frac{3(n + 1)(2n + 1)}{2n^2} \)

e) \( 3 + \frac{9(n + 1)^2}{2n^2} \)
25. If \( f(3) = 2 \) and \( f'(x) \geq 3 \) for \( 2 \leq x \leq 3 \), then the largest possible value of \( f(2) \) is

a) \(-1\)
b) 1
c) \( \frac{1}{3} \)
d) \( \frac{1}{6} \)
e) 0

26. If \( c \) is a number satisfying the conclusion of the Mean Value Theorem when applied to \( f(x) = \ln x \) on \([1, e]\), then \( c = \)

a) \( e - 1 \)
b) \( 1 - e \)
c) \( e \)
d) \( 2e \)
e) \( e - 2 \)
27. The sum of the absolute maximum value and the absolute minimum value of the function \( f(x) = \sin x + \cos x + 1 \) on the interval \([0, \frac{\pi}{2}]\) is

a) \(3 + \sqrt{2}\)
b) \(1 + \sqrt{2}\)
c) \(2 + \sqrt{2}\)
d) \(1 + \frac{1}{\sqrt{2}}\)
e) \(\frac{1}{\sqrt{2}}\)

28. If \((a, b)\) is a point on the hyperbola \(x^2 - y^2 = 4\) which is closest to the point \((0, 1)\), then \(b = \)

a) \(\frac{1}{2}\)
b) \(\frac{1}{4}\)
c) \(1\)
d) \(9\)
e) \(\frac{1}{11}\)