1. If $y = \cos 2x$, then $y^{(99)} =$

   a) $2^{99} \sin 2x$
   b) $2^{99} \cos 2x$
   c) $-2^{99} \sin 2x$
   d) $-2^{99} \cos 2x$
   e) $2^{98} \sin 2x$

2. $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{\sin 2x - \frac{\sqrt{3}}{2}}{x - \frac{\pi}{6}} =$

   a) 1
   b) $\dfrac{1}{2}$
   c) $\dfrac{\sqrt{3}}{2}$
   d) $\sqrt{3}$
   e) 0

3. If $f(x) = \sqrt{1+\sqrt{1+\sqrt{x}}}$, then $f'(9) =$

a) $\dfrac{1}{48\sqrt{3}}$

b) $\dfrac{1}{24\sqrt{3}}$

c) $\dfrac{1}{8\sqrt{3}}$

d) $\dfrac{1}{6\sqrt{3}}$

e) $\dfrac{1}{24}$

4. The number of points on the graph of $x^2 + 3\ x\ y + 9\ y^2 = 3$ where the tangent line is horizontal is

a) 2

b) 1

c) 3

d) 4

e) 0

5. If $f(x) = (1 + 3\ x)^{\ln x}$, then $f'(1) =$

a) $\ln 4$
b) $\ln 2$
c) $\ln 3$
d) 1
e) 0

6. If $y = \tan^{-1}(\sin x) + \cot^{-1}(\csc x)$, then $\dfrac{dy}{dx} =$

a) $\dfrac{2\cos x}{1 + \sin^2 x}$
b) 0
c) $\dfrac{\cos x}{1 + \sin^2 x}$
d) $\dfrac{1}{1 + \csc^2 x}$
e) $\dfrac{1}{1 + \sin^2 x} - \dfrac{1}{1 + \csc^2 x}$

7. A man 2 m tall walks directly away from a street light that is 8 m high at the rate of $\frac{3}{2}$ m/sec. How fast is the length of his shadow changing?

a) $\frac{1}{2}$ m/sec
b) $\frac{1}{3}$ m/sec
c) 3 m/sec
d) $\frac{9}{2}$ m/sec
e) $\frac{3}{2}$ m/sec

8. The linearization of the function $f(x) = \dfrac{1}{1+\sqrt{x}}$ at $x = 1$ is

a) $L(x) = -\frac{x}{8} + \frac{5}{8}$
b) $L(x) = -\frac{x}{2} + 1$
c) $L(x) = \frac{x}{8} + \frac{3}{8}$
d) $L(x) = -\frac{x}{8} - \frac{3}{8}$
e) $L(x) = -\frac{x}{8} + \frac{1}{8}$

9. If $y = \dfrac{(x+2)^2(x-1)^3}{\sqrt{x+1}} + \ln \pi$, then $y'(0) =$

a) 10

b) $10 - \dfrac{5}{2} \ln \pi$

c) 20

d) $20 - 5 \ln \pi$

e) 0

10. Let f and g be differentiable functions and $h(x) = f^2(x+1) - \ln |g(x)| - e^{g(x)}$. If $g(1) = 1, g'(1) = 2, f(2) = 3$ and $f'(2) = 4$, then $h'(1) =$

a) $22 - 2\ e$

b) $20 - 2\ e$

c) $26 - 2\ e$

d) $20 - e$

e) $22 - e$

11. The equation of the tangent line to the graph of $y = \dfrac{\ln x}{x}$ and passes through the origin is

a) $2e\ y = x$
b) $y = 2\ e\ x$
c) $y = 3\ e\ x$
d) $3\ e\ y = x$
e) $y = e\ x$

12. One side of a rectangle is increasing at a rate of 3 cm/sec and the other side is decreasing at a rate of 4 cm/sec. How fast is the area of the rectangle changing when the increasing side is 12 cm long and the decreasing side is 10 cm long?

a) $-18 \quad cm^2/sec$
b) $18 \quad cm^2/sec$
c) $78 \quad cm^2/sec$
d) $12 \quad cm^2/sec$
e) $-12 \quad cm^2/sec$

13. The position function of a particle moving along a line is

$$s(t) = \cos\left(\frac{\pi}{2}t\right) + \sin\left(\frac{\pi}{2}t\right)$$

where t is measured in seconds and s in meters. The total distance traveled by the particle in the interval $[0, 1]$ is

a) $2\sqrt{2} - 2$ meters
b) 2 meters
c) 4 meters
d) $2\sqrt{2} + 2$ meters
e) $2\sqrt{2}$ meters

14. The position function of a particle moving along a line is

$$s(t) = \frac{1}{3}\, t^3 - \frac{3}{2}\, t^2 + 2\, t \quad (0 \le t \le 4).$$

The time interval(s) where the particle is moving backward is (are)

a) $(1, 2)$
b) $(0, 1)$
c) $(0, 1)$ and $(2, 4)$
d) $(1, 4)$
e) $(0, 2)$

15. The height h of a right circular cone is 30 cm whereas the radius r of the cone is 10 cm. There is no error in measurement of the height, but the radius of the cone is known to be accurate to within 0.1 cm. Using differentials, the maximum possible error in computing the volume of the cone is

Hint: Volume of a cone $= \frac{1}{3}\pi r^2 h$

a) $20\ \pi \quad cm^3$
b) $10\ \pi \quad cm^3$
c) $30\ \pi \quad cm^3$
d) $15\ \pi \quad cm^3$
e) $\pi \quad cm^3$

16. The slope of the normal line to the curve $e^y \ln x = e^x \ln y$ at (1,1) is equal to

a) -1
b) 1
c) e
d) $-e$
e) 0

17. The function $f(x) = (3\,x - 2)^{\frac{1}{5}}$ fails to be differentiable at $x = \dfrac{2}{3}$ where it has

a) a vertical tangent
b) a cusp
c) a corner
d) a removable discontinuity
e) an infinite discontinuity

18. If $x = \cos(u^2 + u)$ and $y = \sin(u^2 + u)$, then $\dfrac{dy}{dx}$ at $u = 1$ is

a) $-\cot 2$
b) $-\tan 2$
c) $-\sec 2$
d) $-\csc 2$
e) 0

19. The slope of the tangent line to the graph of $y = 2^{3x} - \log_3 (x+1)^5$ at $x = 1$ is

a) $8 \ln 8 - \dfrac{5}{2 \ln 3}$

b) $4 \ln 8 - \dfrac{5}{\ln 3}$

c) $12 \ln 2 - \dfrac{5}{2 \ln 3}$

d) $3 \ln 2 - \dfrac{5}{2}$

e) $24 \ln 2 - \dfrac{5}{2}$

20. Let f and g be differentiable functions up to the third order and $h(x) = f(x)g(x)$, then $h^{(3)}(x) =$

a) $f'''g + 3f''g' + 3f'g'' + fg'''$

b) $f'''g' + 2f''g'' + f'g'''$

c) $f'''g'''$

d) $f'''g + 2f''g'' + fg'''$

e) $f'''g + fg'''$