

Problem 1: (16 points)) Find the limit if it exists

- a) $\lim_{x \rightarrow \frac{\pi}{2}} \lfloor \sin x \rfloor$
b) $\lim_{x \rightarrow 0} x \cot 2x$
c) $\lim_{x \rightarrow -\infty} \frac{2x-1}{|x|+3}$
d) $\lim_{x \rightarrow \infty} 1 + \frac{\cos 3x}{x}$

Problem 2: (12 points)

- (i) Find all points where the function is continuous $f(x) = \frac{\ln x}{\sqrt{3-x}}$.
- (ii) Show that the equation $x^4 + x^3 + x - 1 = 0$ has a real root.
- (iii) Consider the function $f(x) = \begin{cases} \frac{x-b}{b+1} & \text{if } x \leq 0 \\ x^2 - 2x - 2 & \text{if } x > 0 \end{cases}$

Find all values of b which will make $\lim_{x \rightarrow 0} f(x)$ exists.

Problem 3: (12 points)

- (a) Does the function $f(x) = \sqrt{x^2+1} - x$ have horizontal asymptotes? If yes find it.
- (b) Does the function $f(x) = \frac{1-2x^2}{x+1}$ have oblique asymptotes? If yes find them.
- (c) Does the function $f(x) = \frac{x^2-9}{x^2-2x-15}$ have vertical asymptotes? If yes find them.