Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.

2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.

3. Only materials provided by the instructor can be present on the table during the exam.

4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.

6. Only answers supported by work will be considered. Unsupported guesses will not be graded.

7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.

8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

<table>
<thead>
<tr>
<th>Question</th>
<th>Total Marks</th>
<th>Marks Obtained</th>
<th>Comments</th>
</tr>
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<tr>
<td>1</td>
<td>5+6=11</td>
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<tr>
<td>2</td>
<td>13</td>
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<tr>
<td>3</td>
<td>10</td>
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<tr>
<td>4</td>
<td>4+6=10</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>4+1=5</td>
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<tr>
<td>Total</td>
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1. (5+6=11 points) The loss random variable $X$ has a p.d.f given by

$$f(x) = \frac{1}{150} \quad 0 < x < 150.$$ 

a) Calculate $E[X]$ and $Var(X)$

b) Consider a proportional policy where $I(x) = dx$ $0 < x < 1,$ and a policy-limit policy where $I(x) = \begin{cases} 
  x & x < l \\
  l & x \geq l.
\end{cases}$

Determine $k$ and $l$ such that the pure premium is $P = 12.5$ in each case.
2. (13 points) Consider a portfolio of 64 policies. For each policy, the probability \( q \) of a claim is \( \frac{1}{10} \) and \( B \), the benefit amount given that there is a claim, has a p.d.f.

\[
f(y) = \begin{cases} 
  2 - 3y^2 & 0 < y < 1 \\
  0 & \text{elsewhere.}
\end{cases}
\]

Let \( S \) be the total claims for the portfolio. Using a normal approximation, estimate \( \Pr(S > 3) \).
3. (10 points) Independent random variables $X_k$ for three lives have the discrete probability functions given below

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\Pr(X_2 = x)$</th>
<th>$\Pr(X_3 = x)$</th>
<th>$\Pr(X_4 = x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
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<td>0.1</td>
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</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>0.4</td>
</tr>
</tbody>
</table>

Use a convolution process on the non-negative integer values of $x$ to obtain $F_s(x)$ for $x = 0, 1, 2, \ldots, 6$ where $S = X_1 + X_2 + X_3$. 
4. (4+6=10 points) Using the illustrative life-table provided and under the assumption that deaths are uniformly distributed within each year of age, calculate the following:

a. probability that (70) will die between ages 71 and 73.
b. probability that (70) will die between ages 70.5 and 71.5.
5. (3+4+4=11 points) Consider a modification of De Moivre’s law given by

\[ s(x) = \left(1 - \frac{x}{\omega}\right)^2 \quad 0 \leq x \leq \omega. \]

Calculate
a) \( \mu(x) \)
b) \( s_p_x \)
c) \( \hat{e}_x \)
6. (4+1=5 points) For a select-and-ultimate mortality table with a 3-year select period:

(i)\[
\begin{array}{cccccc}
  x & q_x & q_{x+1} & q_{x+2} & q_{x+3} & x + 3 \\
  60 & 0.09 & 0.11 & 0.13 & 0.15 & 63 \\
  61 & 0.10 & 0.12 & 0.14 & 0.16 & 64 \\
  62 & 0.11 & 0.13 & 0.15 & 0.17 & 65 \\
  63 & 0.12 & 0.14 & 0.16 & 0.18 & 66 \\
  64 & 0.13 & 0.15 & 0.17 & 0.19 & 67 \\
\end{array}
\]

(ii) Wilson was a newly selected life on 01/01/2000.
(iii) Wilson’s age on 01/01/2001 is 61.
(iv) $P$ is the probability on 01/01/2001 that Wilson will be alive on 01/01/2006.

Calculate $P$.
(A) $0 \leq P < 0.43$
(B) $0.43 \leq P < 0.45$
(C) $0.45 \leq P < 0.47$
(D) $0.47 \leq P < 0.49$
(E) $0.49 \leq P \leq 1.00$

Final answer (1 point)
Work shown (4 points)

So Answer is ___

END OF TEST PAPER