Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.

2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.

3. Only materials provided by the instructor can be present on the table during the exam.

4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.

6. Only answers supported by work will be considered. Unsupported guesses will not be graded.

7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.

8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

<table>
<thead>
<tr>
<th>Question</th>
<th>Total Marks</th>
<th>Marks Obtained</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4+5=9</td>
<td>9</td>
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<td>4</td>
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<tr>
<td>5</td>
<td>2+14*0.5=9</td>
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<td>6</td>
<td>4+4=8</td>
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<td>4+4=8</td>
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<td>8</td>
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<td>9</td>
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<tr>
<td>10</td>
<td>4+1=5</td>
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</table>

Total 70

Bonus 6
Extra blank page
1. (4+5=9 points) If \( \mu(x) = \frac{3}{100 - x} - \frac{10}{250 - x} \) for \( 40 < x < 100 \), calculate

a) \( 40 \mu_{50} \)

b) The mode of the distribution of \( X \), the age-at-death.

(Hint: \( f_X(x) \) is concave downwards with negative second derivative).
2. (5 points) Let \( \bar{Z}_x \) denote the present value random variable for a continuously increasing contingent payment contract with benefit \( b_t = 1 + 0.10t \) for failure at time \( t \). Also \( \nu' = (1 + 0.10t)^{-2} \) and the PDF of \( T(x) \), the random variable for time of failure, is given by \( f_{T(x)}(t) = 0.02 \) for \( 0 \leq t \leq 50 \), find the variance of \( \bar{Z}_x \).

3. (5 points) A 10-payment whole life contract of amount 1000 is issued to \( (x) \). The net annual nonlevel premium is 32.88 and the NLP (for NonLevel Premium) benefit reserve at the end of year 9 is 322.87. Given \( i = 0.06 \) and \( q_{x+9} = 0.01262 \), find the value of
   a) \( 10V \)
   b) \( A_{x+10} \).
4. (6 points) A 3-year term insurance issued to \((x)\) has a decreasing failure benefit, paid at the end of the year of failure. The insurance is funded by level benefit premiums. The interest rate is \(i = 0.06\). Calculate the initial reserve (at beginning of year) for the second year, given the following values:

\[
\begin{align*}
    b_1 &= 200 &\quad b_2 &= 150 &\quad b_3 &= 100 \\
    q_x &= 0.03 &\quad q_{x+1} &= 0.06 &\quad q_{x+2} &= 0.09
\end{align*}
\]
Directions: For questions 5 and 6 below, on the basis of the Illustrative Life Table with interest at 6%, consider a 5-year term life insurance of 1000 is issued on a fully discrete basis to each member of a group of \( l_{40} \) persons at age 40.

5. \((2 + 14 \times 0.5 = 9\) marks\) Trace the cash flow expected for this group and, as a by-product, obtain the benefit reserves.

Part of the work is already shown below. Complete the work by filling in the blanks with the correct numbers. (show an example work for one similar entry from the same column)

| Yr \( h \) | Expected Benefit Premiums at Start of Year | Expected Fund at Start of Year | Expected Interest amount | Expected Death Claims | Expected Fund at Year End | Expected Number of Survivors at Year End | \( 1000 \times a_{\overline{5}\!|}^{5\%} \) |
|---------|------------------------------------------|-------------------------------|--------------------------|-----------------------|---------------------------|------------------------------------------|------------------------------------------|
| 1       | 280676.46                                | 16840.59                      | 38498.45                 |                       |                           | 0.4145                                   |                                          |
| 2       | 318394.29                                | 19103.66                      | 276927.10                | 92595.70              | 92299.23                  | 0.6541                                   |                                          |
| 3       | 339632.11                                | 296462.30                     | 63547.74                 | 91981.47              | 91640.50                  | 0.4833                                   |                                          |
| 4       |                                          |                               |                          |                       |                           |                                          |                                          |
| 5       | 321666.67                                | 340973.00                     |                          |                       |                           |                                          |                                          |
6. (4+4=8 points) Consider an insured who has survived to the end of the third policy year. For this insured, evaluate
   a. \( Var[L|K(40) \geq 3] \) directly
   b. \( Var[L|K(40) \geq 3] \) by means of the Hattendorf theorem.
7. (4+4=8 points) For a 5000 4-year term insurance on (50) with return of premiums in case of death with $i = 0.06$, the following are true:

i) the 5000 sum insured benefit is payable at the end of the year of death

ii) the return of premiums is in addition to the 5000 sum insured benefit at the end of the year of death

iii) mortality follows the illustrative life table and

iv) the benefit premiums are accumulated immediately upon receipt at an interest rate $j$.

For this contract, find the net level premium if the premiums are

(a) accumulated without interest (i.e. $j = 0$).

(b) accumulated at the interest rate used in the determination of premiums (i.e. $j = i$).
8. (5+5=10 points) On the basis of the Illustrative Life Table with the assumption of uniform distribution of deaths (UDD) over each year of age and \( i = 0.06 \), we obtained the following:

   i) \( A_{50:20}^1 = 0.13036536 \), \( A_{50:20} = 0.36083889 \), \( a_{50:20} = 11.291832 \),

   ii) \( P_{50:20}^1 = 0.01154510 \), \( P_{50:20} = 0.03195574 \) and

   iii) \( P_{50:20}^{(2)} = 0.032519 \).

Calculate the following for a 20-year endowment insurance issued to (50) with a unit benefit and true semiannual benefit premiums:

a) The benefit reserve at the end of the tenth year if the benefit is payable at the end of the year of death.

b) The benefit reserve at the end of the tenth year if the benefit is payable at the moment of death.
9. (5 points) Let 0\( L(A_x) \) denote the present value of loss at issue for a fully continuous whole life contract issued to \( (x) \). Given the following values, find the value of 20\( V(A_x) \):

\[
\text{Var}[0L(A_x)] = 0.20 \quad 2\bar{A}_x = 0.30 \quad \bar{A}_{x+20} = 0.70.
\]

10. (4+1=5 marks) For a special 3-year term life insurance on (50), you are given:
   (i) The death benefit of 1000 is paid at the end of the year of death
   (ii) The annual effective rate of interest is 4%
   (iii) The benefit premium in year 1 is 1000\( A_{50} \)
   (iv) The benefit premiums in years 2 and 3 are equal
   (v) The mortality table has the following values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( q_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.05</td>
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<tr>
<td>51</td>
<td>0.06</td>
</tr>
<tr>
<td>52</td>
<td>0.07</td>
</tr>
<tr>
<td>53</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Calculate the benefit reserve at the end of year 2.

a) 0
b) 4.856
c) 5.001
d) 5.051
e) 5.252
f) 5.316

Hence the answer is: _____

END OF TEST PAPER
11. (**Bonus** =6 points) Consider a portfolio of 81 policies. For each policy, the probability $q$ of a claim is $1/10$ and $B$, the benefit amount given that there is a claim, has a p.d.f.

$$f(y) = \begin{cases} 
2 - 2y & 0 < y < 1 \\
0 & \text{elsewhere}
\end{cases}$$

Let $S$ be the total claims for the portfolio. Using a normal approximation, estimate $\Pr(S > 3)$. 