Department of Mathematics and Statistics
Semester 141

AS388
Final Exam
Wednesday December 24, 2014

Exam Version #1

Name: ______________________________ ID #: ____________________

- Check the appropriate answer.

- Questions not answered will not count even if the detailed solution is shown.

- The exam is 30 questions. If you score 24 and above you get a full score of 30.

- Duration of the exam is 3 hours.

1) Y is a continuous random variable with density $f(y) = \begin{cases} |y| & 1 \leq \quad y \leq 1 \\ 0 & otherwise \end{cases}$, then $E(|Y|)$ is

   a) 0
   b) $\frac{1}{6}$
   c) $\frac{1}{3}$
   d) $\frac{1}{2}$
   e) $\frac{2}{3}$

2) As part of the underwriting process for insurance, each prospective policyholder is tested for diabetes. Let Y represent the number of tests completed when the first person with diabetes pressure is found. The expected value of Y is 8. The probability that the fourth person tested is the first one with diabetes is

   a) 0.000
   b) 0.050
   c) 0.064
   d) 0.083
   e) 0.166

3) If Y has a normal distribution with mean 1 and variance 4, then $P(Y^2 - 4Y \leq 0)$ is

   a) < 0.15
   b) [0.15 , 0.35)
   c) [0.35 , 0.55)
   d) [0.55 , 0.75)
   e) > 0.75

4) X and Y have a bivariate normal distribution with the following joint pdf

$f(x, y) = \frac{0.3125}{\pi} e^{-0.71825(x^2 - 0.6xy + 0.25y^2)}$.The marginal distributions of X and Y are both normal with mean 0, but X has variance of 1, and Y has variance of 4. Then the coefficient of correlation between $X+Y$ and $X-Y$ is

   a) < – 0.6
   b) [– 0.6 , – 0.2)
   c) [– 0.2 , 0.2)
   d) [0.2 , 0.6)
   e) > 0.6

5) Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} c(y-x) & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$$

The mean of the marginal distribution of X is

a) $\frac{1}{8}$
b) $\frac{1}{4}$
c) $\frac{3}{8}$
d) $\frac{1}{2}$
e) $\frac{5}{8}$

6) Statistics show that if a team has won 3 games and lost 1 game out of the first 4 games during a "best of 7" series, that team has an 80% chance of winning the series. Statistics also show that if a team has won 3 games and lost 1 game out of the first 4 games and then loses the 5^{th} game, that team has a 65% chance of winning the series. The probability that a team that has won 3 games and lost 1 game out of the first 4 games will win the next game is:

a) $\frac{2}{7}$
b) $\frac{3}{7}$
c) $\frac{4}{7}$
d) $\frac{5}{7}$
e) $\frac{6}{7}$

7) The number of goals scored per game by team *A*, has a geometric distribution, $X_A = 0, 1, 2, \ldots$ with mean of 3.5. The number of goals scored per game by team *B*, has a geometric distribution, $X_B = 0, 1, 2, \ldots$ with mean of 3.0. Assuming that X_A and X_B are independent, the probability that team *B* wins the game by at least 2 goals is

a) 0.1
b) 0.15
c) 0.2
d) 0.25
e) 0.3

8) The random variable Y is Binomial(2, 0.5) with probability p, and Binomial(4, 0.5) with probability 1 – p. Then $P(Y = 2)$ is

a) $0.125\, p^2$
b) 0.375 + 0.125p
c) 0.375 + 0.125 p^2
d) 0.375 – 0.125 p^2
e) 0.375 – 0.125p

9) A fair coin is independently tossed 100 times. The number of heads tossed is Y. It is desired to find the smallest integer value k which satisfies $P(50 - k \leq Y \leq 50 + k) \geq 0.95$. By applying the normal approximation with integer correction to the distribution of Y, k is

a) 6
b) 7
c) 8
d) 9
e) 10

10) The loss random variable X has an exponential distribution with mean θ > 0. An insurance company pays Y,

where $Y = \begin{cases} \frac{X}{2} & if\ X \leq \theta \\ X & if\ X > \theta \end{cases}$

Then *E(Y)* is:

a) $\frac{\theta}{2}(1 + e^{-1})$
b) $\frac{\theta}{2}(1 + 2e^{-1})$
c) $\frac{\theta}{2}(1 - e^{-1})$
d) $\frac{\theta}{2}(1 - 2e^{-1})$
e) $\theta(1 - e^{-1})$

11) If X has a continuous distribution on the interval [0 , 1] and the conditional distribution of Y given is a continuous uniform distribution on the interval [x , 2], then *E(Y)* is

a) $\frac{3}{4}$
b) 1
c) $\frac{5}{4}$
d) $\frac{6}{4}$
e) $\frac{7}{4}$

12) A loss random variable X has the continuous uniform distribution on (0, 100). An insurance policy on the loss pays the full amount of the loss if the loss is less than or equal to 40. If the loss is above 40 but less than or equal to 80, then the insurance pays 40 plus one-half of the loss in excess of 40. If the loss is above 80, the insurance pays 60. If Y denotes the amount paid by the insurance when a loss occurs, then the variance of Y is

a) $\frac{1020}{3}$
b) $\frac{1040}{3}$
c) $\frac{1060}{3}$
d) $\frac{1080}{3}$
e) $\frac{1120}{3}$

13) A survey found out that 80% of the residents of a city had a driver's license or owned a bicycle, or both, one third of those who had a driver's license also owned a bicycle, one half of those who owned a bicycle also had a driver's license. Of those surveyed who did not have a bicycle, the fraction who did not have a driver's license is:

a) $\frac{1}{3}$
b) $\frac{4}{9}$
c) $\frac{5}{9}$
d) $\frac{2}{3}$
e) $\frac{7}{9}$

14) A large class has two term tests and a final exam. Students are allowed to drop the course before the first term test. Class records for past year show the following:

80% of students pass the first test.

30% of students who fail the first term test drop the course before the second test.

10% of student who pass the first term test drop the course before the second test

90% of students who pass the first term test and take the second test pass the second test.

80% of students who fail the first term test and take the second test pass the second test

50% of students who fail the second term test drop the course before the final exam.

None of the students who pass the second term test drop the course before the final exam.

The fraction of students who drop the course is

a) < 0.05
b) [0.05, 0.1)
c) [0.1, 0.15)
d) [0.15, 0.20)
e) > 0.20

15) In the 2006 World Cup of soccer, according to an onlilne ranking service, Brazil, England and Germany are the three most highly ranked teams to win the tournament. A survey of soccer fans asks the fans to rank from most likely to least likely the chance of each of those country's teams winning the world cup. The survey found the following:

Two thirds of those who ranked Germany first ranked Brazil second

One seventh of those who did not rank Germany first ranked Brazil second

30% of those surveyed ranked Brazil second.

Of those surveyed who ranked Brazil second, the proportion that ranked Germany third is

a) $\frac{1}{4}$
b) $\frac{1}{3}$
c) $\frac{1}{2}$
d) $\frac{2}{3}$
e) $\frac{3}{4}$

16) An urn has 6 identically shaped balls. 4 of the balls are white and 2 of the balls are blue. A ball is chosen at random from the urn and replaced with a white ball. The procedure is done repeatedly. The probability that after the *n*-th application of this procedure there is exactly one blue ball in the urn is:

a) $\left(\frac{5}{6}\right)^n$
b) $\left(\frac{5}{6}\right)^n - \left(\frac{2}{3}\right)^n$
c) $2\left[\left(\frac{5}{6}\right)^n - \left(\frac{2}{3}\right)^n\right]$
d) $\frac{1}{2}\left(\frac{5}{6}\right)^n$
e) $\frac{1}{2}\left[\left(\frac{5}{6}\right)^n - \left(\frac{2}{3}\right)^n\right]$

17) X has a Poisson distribution with mean 1. Y is defined in the following way:

$P(Y = 0) = \alpha, 0 < \alpha < 1$, and $P(Y = x) = c.P(X = x)\ for\ x = 1, 2, \ldots$

The value of *c* that makes *Y* satisfy the requirements of being a random variable is:

a) $\frac{1-\alpha}{1-e^{-\alpha}}$
b) $\frac{1-\alpha}{e^{\alpha}-1}$
c) $\frac{1-\alpha}{1-e^{-1}}$
d) $\frac{1-\alpha}{e-1}$
e) $\frac{\alpha}{e-1}$

18) A hockey team has two suppliers of sticks, supplier A and supplier B. The team gets equal numbers of sticks from each supplier, and since the team logo is branded on every stick, after the sticks are delivered, it is not possible to tell what supplier provided any particular stick. The estimate that on average, 10% of the sticks come from supplier A are defective and 20% of the sticks from supplier B are defective. A team player examines 10 sticks from a recent shipment and finds 2 defective sticks. The probability that the supplier of those sticks is supplier A is

a) < 0.11
b) [0.11, 0.22)
c) [0.22, 0.33)
d) [0.33, 0.44)
e) [0.44, 0.50)

19) A sports team is considering a one-time charitable program of making a donation to children's hospital. The donation will be related to how many goals they score in their next game. The team statistician has determined that the number of goals scored in a game has a Poisson distribution with a mean of 3. The team is planning to donate \$ C for each goal they score up to a maximum of 3 goals. The value of C that would make the team's expected donation for a game to be \$5000 is

a) < 1500
b) [1500, 2000)
c) [2000, 2100)
d) [2100, 2200)
e) [2200, 2300)

20) Y has a distribution which is partly discrete and partly continuous. $Y = 1$ with probability p. On the interval (0, 1) Y has the density $\frac{1-p}{2}$, and on the interval (1, 2) Y has the density $\frac{1-p}{2}$. The variance of Y is:

a) $\frac{1-p}{3}$
b) $\frac{2-p}{3}$
c) $\frac{1-p}{2}$
d) $\frac{2-p}{2}$
e) $\frac{1+p}{2}$

21) X has the pdf $f(x) = \begin{cases} x - \frac{x^2}{2} & 0 < x \leq 1 \\ \frac{x^2}{2} - x + 1 & 1 < x < 2 \end{cases}$. If $Y = X^2$, then $F_Y(2)$ is

a) 0.22
b) 0.33
c) 0.44
d) 0.55
e) 0.66

22) X_1 has a Binomial distribution with mean 2 and variance 1, X_2 has a Poisson distribution with variance 2, X_1 and X_2 are independent. Then $P(X_1 + X_2 < 3)$ is

a) $\frac{11}{16}e^{-2}$
b) $\frac{15}{16}e^{-2}$
c) $\frac{19}{16}e^{-2}$
d) $\frac{23}{16}e^{-2}$
e) $\frac{27}{16}e^{-2}$

23) X has pdf $f(x) = \begin{cases} 0 & x < 0 \\ x & 0 < x < 1 \\ 0 & x > 1 \end{cases}$, also $P(X = 0) = a$ and $P(X = 1) = b$.

The value of a that maximizes the variance of X lies in the interval

a) [0, 0.1)
b) [0.1, 0.2)
c) [0.2, 0.3)
d) [0.3. 0.4)
e) [0.4, 0.5)

24) If $P(A|B) = P(A'|B') = 0.4$ and $P(A) = 0.5$, then $P(B) =$

a) 0.4
b) 0.5
c) 0.6
d) 0.7
e) 0.8

25) A loss random variable is uniformly distributed on the interval (0, 2000). An insurance policy on this loss has an ordinary deductible of 500 for loss amounts up to 1000. If the loss is above 1000, the insurance pays half of the loss amount. The standard deviation of the amount paid by the insurance when a loss occurs is

a) < 250
b) [250, 275)
c) [275, 300)
d) [300, 325)
e) [325, 350)

26) X and Y have the joint pdf

$$f(x,y) = \begin{cases} \dfrac{2x+y}{12} & 0 \le x \le 2, \quad 0 \le y \le 2 \\ 0 & otherwise \end{cases}$$

Then $P(X+Y \ge 2|X \le 1)$ is

a) $\frac{1}{4}$
b) $\frac{3}{8}$
c) $\frac{1}{2}$
d) $\frac{5}{8}$
e) $\frac{6}{8}$

27) X has pdf $f(x) = \begin{cases} ax+b & 0 < x < 2 \\ 0 & otherwise \end{cases}$, and the median if X is 1.25. Then the variance of X lies in the interval

a) (0, 0.05)
b) [0.05, 0.15)
c) [0.15, 0.25)
d) [0.25, 0.35)
e) [0.35, 0.45)

28) In the game of roulette, a wheel with 38 equally likely spots is spun, and a ball is droped at random into one of the 38 spots. The 38 spots are numbers 1 to 36 along wih 0 and 00. On a spin of the wheel, a gambler can bet that the ball will drop into a specified spot. If the ball does drop into that spot, the gambler gets back the amount he bet plus 36 times the amount that he bet. If that spot does not turn up, the gambler loses the amount bet. A gambler can also bet that the outcome of the spin will be even. If the ball drops into an even number spot from 2 to 36, the gambler gets back his bet plus an amount equal to the amount that he bet (the bet is lost if the spot is 0 or 00). On every spin, Gambler 1 always bets 1 that the ball will drop in the sport with the number 1, and Gambler 2 always bets 1 that the ball will drop into an even numbered spot. If X_1 denotes the net profit of Gambler 1 after *n* spins, and X_2 denotes the net profit of Gambler 2 after the *n* spins, then $E(X_2 - X_1)$ equals

a) $-\frac{n}{19}$
b) $-\frac{n}{38}$
c) 0
d) $\frac{n}{38}$
e) $\frac{n}{19}$

29) A loss random variable has a Poisson distribution with mean μ. An insurance policy on the loss has a policy limit of 1. The expected insurance payment when a loss occurs is 0.8892. Let E be the expected insurance payment when a loss occurs for a policy on the same loss variable if the policy limit is 2. Then E lies in the interval

a) < 0.35
b) [0.35, 0.7)
c) [0.7, 1.05)
d) [1.05, 1.40)
e) [1.40, 1.75)

30) An insurer has two lines of business: auto insurance and home fire insurance. People with home fire insurance policy can add flood insurance coverage, but only if the policy already has fire coverage. You are given the following information about the insurer's customers:

80% of all customers have an auto insurance policy

40% of all customers have a fire insurance policy

25% of customers with an auto insurance policy also have a fire insurance policy

50% of customers with a fire insurance policy also have flood insurance.

50% of customers with flood insurance coverage also have auto insurance.

Of the insurer's customers that have fire insurance, the fraction that have neither auto insurance nor flood insurance coverage is

a) 0.05
b) 0.10
c) 0.15
d) 0.20
e) 0.25