

Name: _____

Sr#: _____

Q1. Let C be the curve defined by the parametric equations

$$x = 2 + 3\cos(t), \quad y = 1 + 3\sin(t), \quad 0 \leq t \leq \pi$$

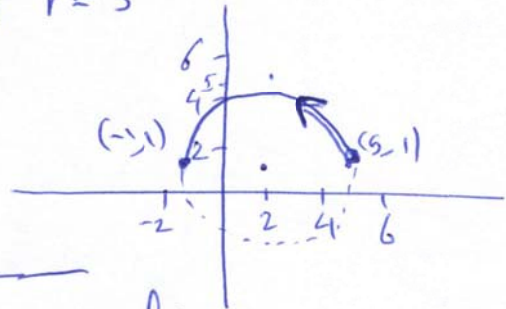
- Find Cartesian equation for C
- Sketch the graph and indicate on it the direction in which C traced.
- Find the area of the surface generated by revolving C about the x -axis.

$$a) \left(\frac{x-2}{3}\right)^2 + \left(\frac{y-1}{3}\right)^2 = \cos^2 t + \sin^2 t = 1 \Rightarrow (x-2)^2 + (y-1)^2 = 9$$

b) Circle center $(2, 1)$ radius $r = 3$

$$t = 0 \rightarrow (5, 1), \quad t = \frac{\pi}{2} \rightarrow (2, 4)$$

$$t = \pi \rightarrow (-1, 1)$$



$$c) A = 2\pi \int_0^{\pi} (1 + 3\sin t) \sqrt{9\sin^2 t + 9\cos^2 t} dt$$

$$= 2\pi \int_0^{\pi} (1 + 3\sin t) 3 dt$$

$$= 6\pi [t - 3\cos t]_0^{\pi} = 6\pi [\pi - 3 - (0 - 3)]$$

$$= \cancel{12\pi} = 6\pi^2$$

Q2. Consider the curve C whose equation in polar coordinate is given by

$$r^2 = 4\cos(2\theta)$$

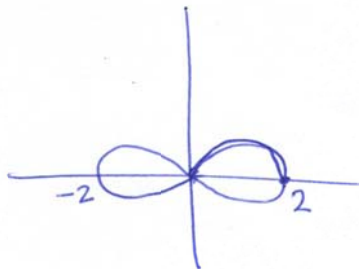
- Find the symmetric in the graph of C (if any).
- Sketch the graph in xy -plane.
- Find the area of the region enclosed by C.

x-axis $(r, -\theta) \rightarrow r^2 = 4\cos(2(-\theta)) \rightarrow r^2 = 4\cos 2\theta \leftarrow (r, 0)$

y-axis $(r, \pi - \theta) \rightarrow r^2 = 4\cos(2(\pi - \theta))$
 $r^2 = 4\cos(2\pi - 2\theta) = 4\cos 2\theta \leftarrow$

origin \curvearrowright

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$
r	2	$2\sqrt{3}$	$\frac{4}{\sqrt{2}}$	2	0



~~A = \dots~~ $A = 4 \int_0^{\frac{\pi}{4}} 4\cos 2\theta \, d\theta$
 $= 8 \left[\frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{4}} = 4 [1 - 0] = 4$