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King Fahd University of Petroleum and Minerals

Quiz 1 Math 202-Semester 131 Duration 25 minutes

Full Name:

ID:

Q 1. Determine the order and state whether the following ODEs are linear or nonlinear (Give a brief justification).

a) $\frac{d^2x}{dt^2} + \boxed{(1+\sqrt{x})\frac{dx}{dt}} = \sin t$: Order 2, Nonlinear because of the term $(\sqrt{x}+1)\frac{dx}{dt}$
(x is the dependent variable & t is the independent variable here)

b) $x\,y''' + (1-x)y' = e^x$: order 3. Linear

c) $\frac{d^2u}{dr^2} + \sin(r)\boxed{u\,\frac{du}{dr}} = r^2$; Order 2. Non linear because of the term $u\frac{du}{dr}$

d) $\underline{\underline{y'^2}} = \underline{\underline{\cos y}}$: order 1. Nonlinear because of the term $(y')^2$ & $\cos y$

Q 2.

a- Verify that $y = \tan(x+C)$ is a one-parameter family of solutions of: $y' = 1+y^2$.

b- Find a solution of $y' - y^2 = 1$ such that, it is passing through the point $(0,1)$. Then find the largest interval of definition of the obtained solution.

a) $y = \tan(x+c) \Rightarrow \begin{cases} y' = \sec^2(x+c) \\ 1+y^2 = 1+\tan^2(x+c) = \sec^2(x+c) \end{cases}$

so $y' = 1+y^2$ & $\therefore$ $y = \tan(x+c)$ is a solution of $y' = 1+y^2$.

b) From (a), $y = \tan(x+c)$ is a solution of $y'-y^2 = 1$. Now, we want to find c such that $y(0) = 1$. that is, $1 = \tan(c) \Rightarrow c = \frac{\pi}{4}$ (tan has an inverse on $(-\frac{\pi}{2}, \frac{\pi}{2})$)

so $y = \tan(x+\frac{\pi}{4})$ is a solution of $y'-y^2 = 1$, $y(0) = 1$. For the domain of definition,

$-\frac{\pi}{2} < x+\frac{\pi}{4} < \frac{\pi}{2} \Rightarrow -\frac{3\pi}{4} < x < \frac{\pi}{4}$ $\therefore$ $(-\frac{3\pi}{4}, \frac{\pi}{4})$ is the interval of definition. Noting that $0 \in (-\frac{3\pi}{4}, \frac{\pi}{4})$.

Because tan has an inverse for on $(-\frac{\pi}{2}, \frac{\pi}{2})$

Q 3. Find an explicit solution and the corresponding interval of definition of the IVP:

$$(\sqrt{x}+x)dy-(\sqrt{y}+y)dx=0 \quad \text{with} \quad y(0)=1. \qquad (A)$$

We rewrite (A) as: $\dfrac{dy}{\sqrt{y}+y} = \dfrac{dx}{\sqrt{x}+x}$ ← separable.

⇓ integrate

$$\int \frac{dy}{\sqrt{y}+y} = \int \frac{dx}{\sqrt{x}+x}$$

$$\underline{\int \frac{dx}{\sqrt{x}+x}}$$

Let $\sqrt{x}=w \Rightarrow \dfrac{dx}{2\sqrt{x}} = dw$. so

$$\int \frac{dx}{\sqrt{x}+x} = \int \frac{2\sqrt{x}\,dw}{w+w^2} = 2\int \frac{dw}{1+w} = 2\ln|1+w| + C = 2\ln(1+\sqrt{x}) + C$$

$(x \geq 0)$

Similarly, $\displaystyle\int \frac{dy}{\sqrt{y}+y} = \ln(1+\sqrt{y}) + C.$

$\therefore \ln(1+\sqrt{y}) = \ln(1+\sqrt{x}) + C \Rightarrow 1+\sqrt{y} = Ce^{\ln(1+\sqrt{x})} = C(1+\sqrt{x})$

$y(0)=1 \Rightarrow 1+1 = C(1+0) \Rightarrow \boxed{C=2}$. Thus,

$1+\sqrt{y} = 2(1+\sqrt{x})$ is an implicit solution of (A)

⇓

$y = (2\sqrt{x}+1)^2$ " " explicit solution of (A).

The interval of definition is $[0,\infty)$