Name: ___________________________   ID#: ________________
Section/Instructor: _________________   Serial #: ________________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

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<th>Question #</th>
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Q:1 (10 points) Show that the Fourier transform of \( f(t) = \begin{cases} \cos(at) & |t| < 1 \\ 0 & |t| > 1 \end{cases} \) is 
\[ F(w) = \frac{\sin(w - a)}{w - a} + \frac{\sin(w + a)}{w + a}. \]
Q:2a (8 points) Use partial fractions to invert the Fourier transform:

\[ F(w) = \frac{1}{(1 + w)(1 + 2iw)^2}. \]

Q:2b (8 points) Use the fact that \( F[e^{-3t}H(t)] = \frac{1}{3 + iw} \) and Parseval’s equality to show that

\[ \int_{-\infty}^{\infty} \frac{dx}{9 + x^2} = \frac{\pi}{3}. \]
Q:3 (12 points) Evaluate by Fourier transform \( e^{-t}H(t) \ast e^{t}H(-t) \).
Q:4 (14 points) Solve the Sturm- Liouville problem:

\[ y'' + \lambda y = 0, \quad y(0) + y'(0) = 0, \quad y(\pi) + y'(\pi) = 0. \]
Q:5 (12 points) The Sturm-Liouville problem $y'' + \lambda y = 0, \quad y'(0) = y'(L) = 0$ has the eigen function solutions $y_0(x) = 1$ and $y_n(x) = \cos\left(\frac{n\pi x}{L}\right)$. Find the eigenfunction expansion for $f(x) = x$ using these eigenfunctions.
Q:6 (12 points) Find the expansion with five nonvanishing coefficients in Legendre polynomials of the function: 
\[ f(x) = \begin{cases} 
1 & 0 < x < 1 \\
0 & -1 < x < 0 
\end{cases} \]
Q:7 (14 points) Let \( A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix} \)

(a) Find the eigenvalues of \( A \)

(b) Find an eigenvector corresponding to \( \lambda = -4 \),
Formula Sheet

If $f(t)$ is a function with Fourier transform $F(w)$, then

(I) $F[f(t - \tau)] = e^{-i\omega \tau} F(w)$

(II) $F[f(kt)] = F(w/k)/|k|$, $k$ is a scalar

(III) $F[F(t)] = 2\pi f(-w)$

(IV) $F[f^{(n)}(t)] = (i\omega)^n F(w)$

(V) $F[f(t)e^{i\omega_0 t}] = F(w - w_0)$