KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Department of Mathematics & Statistics

Math 514 Advanced Methods of Applied Mathematics

Final Term Exam Term 141

Time Allowed 3 Hours

Name__________________ ID # _______

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Important Note

Write clearly and show all work.

Instructor: F. D. Zaman
Q1) Solve the following boundary value problem using an appropriate integral transform.

\[ u_{xx} + u_{yy} = 0, \quad 0 < x < \infty, \quad 0 < y < \infty \]

\[ u(0, y) = a, \quad 0 < y < \infty \]

\[ u(x, 0) = 0, \quad 0 < x < \infty \]

\[ \Delta u \to 0 \text{ as } r \to \infty \]

where a is a constant.
Q2) Define the Hankel transform pair stating appropriate conditions. Show that

\[ H_0 \left\{ \frac{1}{r} \frac{d}{dr} \left( r \frac{df}{dr} \right) \right\} = -\alpha^2 H_0(f). \]

Solve the biharmonic problem

\[ \nabla^4 u(r, z) = 0, \quad 0 \leq r < \infty, \quad z > 0 \]

\[ u(r, 0) = f(r), \quad 0 \leq r < \infty \]

\[ \frac{\partial u}{\partial z} = 0 \quad \text{on} \quad z = 0, \quad 0 \leq r < \infty \]

\[ u(r, z) \to 0 \quad \text{as} \quad r \to \infty \]
Q 3) Solve singular integral equation using the Wiener Hopf technique

\[ u(x) = 3 - 3 \int_0^\infty e^{-|x-\xi|} u(\xi) d\xi, \quad 0 < x < \infty. \]
Q4) Find an asymptotic series representation of the integral

\[ \int_{x}^{\infty} t^{-1} e^{-t} \, dt, \text{ for large } x. \]

Show that the series is divergent in the usual sense. Show that this series is asymptotic for large value of \( x \).
Q 5) Using the procedure based upon the Laplace method, find an asymptotic representation of

\[ \Gamma(x + 1) = \int_0^\infty e^{-t^x} dt \quad \text{for large } x. \]

(Hint: Put \( t^x = e^{x \ln t} \).)

Obtain an approximation of \( n! \) for large \( n \).