Q:1 Using tree method, develop a general third order 3-stage method. Make a table of trees up to order 3 and write $\Phi(t)$ and $\gamma(t)$ for each tree. Solve the order conditions for $b_i = 0$ and $c_1 = 1$. Also write the corresponding numerical scheme.

Q:2 Develop a Taylor method of order 4 for the problem $y' = y - x^2 + 1, \ y(0) = 0.5$.

Use $h = 0.5$ and simplify the numerical scheme.

Q:3 Find the coefficients $\beta_i$ for the Adom-Bashforth method $y_n = y_{n-1} + \sum_{i=1}^{3} \beta_i f(x_{n-i}, y_{n-i})$

Q:4 Show that $e_n \approx \frac{1}{12} h^3 y''''(x_{n-1})$ for the method $y_n = y_{n-1} + \frac{h}{2} [f(x_{n-1}, y_{n-1}) + f(x_n, y_n)]$.

Write order of the method.

Q:5 Find $\sigma(t), \ \gamma(t), \ \alpha(t), \ \beta(t)$ for the following trees (i) $[[\tau^2]\tau^3]$, (ii) $[[\tau^3]\tau^2]$, (iii) $[[[\tau^2]\tau]]$