

Dept of Mathematics and Statistics  
King Fahd University of Petroleum & Minerals

AS475: Survival Models for Actuaries  
Dr. Mohammad H. Omar  
Major 3 Exam Term 142 FORM A  
Thursday May 7 2015  
1.30pm-3.00pm

Name \_\_\_\_\_ ID#: \_\_\_\_\_ Serial #: \_\_\_\_\_

**Instructions.**

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	$3+4=7$		
2	7		
3	$4+4=8$		
4	$4+2+4+2=12$		
5	$2+2+2=6$		
Total	40		

Extra blank page

1. (3+4=7 points) For the data below and a Weibull model (with  $\tau = 0.95$  and  $\beta = 854$ ),  
 a) complete the blank values below. Be sure to show how you calculate the blank values.

$x$	$F_n(x-)$	$F_n(x)$	$F^*(x)$	Max Difference
82	0	0.0526	0.1023	0.1023
115	0.0526	0.1053	0.1383	0.0857
126			0.1499	0.0446
155		0.2105	0.1794	0.0312
161	0.2105	0.2632	0.1853	0.0779
243	0.2632	0.3158	0.2614	0.0544
294	0.3158	0.3684	0.3045	0.0639
340	0.3684	0.4211	0.3409	0.0801
384	0.4211	0.4737	0.3737	0.0999
457	0.4737	0.5263	0.4243	0.1020
680	0.5263	0.5789	0.5531	0.0268
855	0.5789	0.6316	0.6325	0.0536
877	0.6316	0.6842	0.6414	0.0428
974	0.6842	0.7368	0.6779	
1193	0.7368	0.7895	0.7469	0.0426
1340	0.7895	0.8421	0.7844	0.0577
1884	0.8421	0.8947	0.8800	0.0379
2558	0.8947	0.9474		
3476	0.9474	1.0000	0.9775	0.0301

- b) Use the Kolmogorov-Smirnov test (with  $\alpha=0.05$ ), to determine if the Weibull model is appropriate.

2. (7 points) One hundred and fifty policyholders were observed from the time they arranged a viatical settlement (a sale of policyholder's life insurance to a third party for more than its surrender value but less than the death benefit) until their death . No observations were censored. There are 21 deaths in the first year, 27 deaths in the second year, 39 deaths in the third year, and 63 deaths in the fourth year. The survival model

$$S(t) = 1 - \frac{t(t+1)}{20}, \quad 0 \leq t \leq 4,$$

is being considered. At a 5% significance level, conduct the chi-square goodness-of-fit test.

3. (4+4=8 points) Provided below are computer results for fitting the interaction model on the vets.dat data which contains  $n = 137$  data points. The data contains the following variables:

Column	Variable	Column	Variable
1	Treatment (standard=1, test=2)	7	Performance status (0=worst, ..., 100=best)
2	Cell type 1 (large=1, other=0)	8	Disease Duration (months)
3	Cell type 2 (adeno=1, other=0)	9	Age
4	Cell type 3 (small=1, other=0)	10	Prior Therapy (0=None, 10=some)
5	Cell type 4 squamous=1, other=0)	11	Status (0=censored, 1=died)
6	Survival time (days)		

The PSbin variable is either high (if Performance status is 60 or above) or low (if performance status is below 60). In the following print-out the variable  $Z_1^*$  denotes the small cell variable and the variable  $Z_2^*$  denotes the PSbin variable. The variable  $DDZ_1^*$  denotes the product of  $Z_1^*$  with disease duration, and other product terms are defined similarly. The stratification variable  $SZ^*$  is a combination of  $Z_1^*$  and  $Z_2^*$  and consist of 4 categories.

**Stratified Cox regression Analysis time t: survt**

	Coef.	Std Err	$p >  z $	Haz.Ratio	95% Conf. Interval	
Treatment	0.381	0.428	0.374	1.464	0.632	3.389
Dis. Durat.	0.015	0.021	0.469	1.015	0.975	1.057
Age	0.000	0.017	0.994	1.000	0.968	1.033
Pr. Therapy	0.023	0.041	0.571	1.023	0.944	1.109
$DDZ_1^*$	-0.029	0.024	0.234	0.971	0.926	1.019
Age $Z_1^*$	-0.055	0.037	0.135	0.946	0.880	1.018
$PTZ_1^*$	0.043	0.075	0.564	1.044	0.901	1.211
$DDZ_2^*$	0.025	0.032	0.425	1.026	0.964	1.092
Age $Z_2^*$	0.001	0.024	0.956	1.001	0.956	1.049
$PTZ_2^*$	-0.078	0.054	0.152	0.925	0.831	1.029
$DDZ_1Z_2^*$	-0.071	0.059	0.225	0.931	0.830	1.045
Age $Z_1Z_2^*$	0.084	0.049	0.084	1.088	0.989	1.196
$PTZ_1Z_2^*$	-0.005	0.117	0.963	0.995	0.791	1.250
$trZ_1^*$	0.560	0.732	0.444	1.751	0.417	7.351
$trZ_2^*$	-0.591	0.523	0.258	0.554	0.199	1.543
$trZ_1Z_2^*$	-0.324	0.942	0.731	0.723	0.114	4.583
No. of subjects = 137		Log likelihood = -335.591		Stratified by $SZ^*$		

a) Use the above computer results to state the form of the **estimated hazard model** for each of the four strata of the stratification variable  $SZ^*$ .

b) For each strata, also compute the **hazard ratio** for the treatment effect adjusted for *disease duration*, *age*, and *prior therapy*.

4. (4+2+4+2=12 points). The following is an edited print out of the results obtained by fitting extended Cox model containing two heaviside functions on the variable Sex:

**Time Dependent Cox regression Analysis:**

Analysis time_t: survt	Coef.	Std Err	$p >  z $	Haz.Ratio	95% Conf. Interval	
<i>log WBC</i>	1.567	0.333	0.000	4.794	2.498	9.202
<i>Rx</i>	1.341	0.466	0.004	3.822	1.533	9.526
0-15 wks	0.358	0.483	0.459	1.430	0.555	3.682
15+ wks	-0.182	0.992	0.855	0.834	0.119	5.831

No. of subjects = 42

Log likelihood = -71.980

Use the above computer results, adjusted for *log WBC* and the time-dependent *Sex* variables, to

- carry out a test of hypothesis on the treatment variable *Rx*,
- estimate the hazard ratio on the treatment variable *Rx*,
- obtain 95% confidence interval for the treatment effect *Rx* and
- provide your conclusion regarding the treatment effect *Rx*.

5. (2+2+2=6 points) The following analysis contains an example of the exponential model using the remission data with treatment status (coded  $TRT = 1$  for the experimental treatment and  $TRT = 0$  for the placebo). The output for both the AFT and PH forms of the model are presented.

<b>Exponential regression</b>				
<b>accelerated failure time form</b>				
$\lambda = \exp[-(\alpha_0 + \alpha_1)TRT]$				
<u>_t</u>	<b>Coef.</b>	<b>Std Err</b>	$z$	$p >  z $
trt	1.527	0.398	3.83	0.00
_cons	2.159	0.218	9.90	0.00

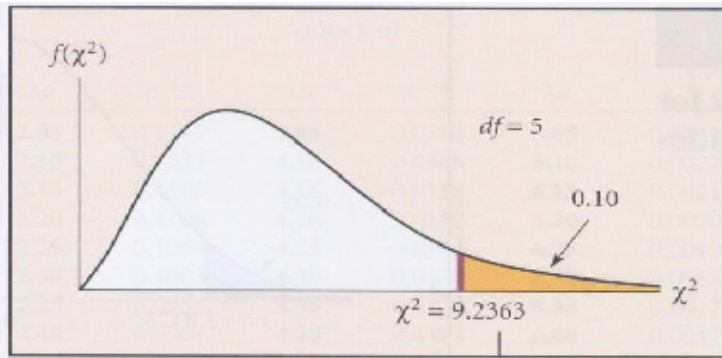
<b>Exponential regression log</b>				
<b>relative hazard form</b>				
$\lambda = \exp(\beta_0 + \beta_1 TRT)$				
<u>_t</u>	<b>Coef.</b>	<b>Std Err</b>	$z$	$p >  z $
trt	-1.527	0.398	3.83	0.00
_cons	-2.159	0.218	-9.90	0.00

- Find the median survival time (in weeks) for the treated (TRT=1) and placebo (TRT=0) groups.
- Find the acceleration factor  $\hat{\gamma}$ .
- In this exponential model, provide the relationship between the coefficients  $\alpha_j$  and  $\beta_j$ .

END OF TEST PAPER

**APPENDIX G**

**Values of  $\chi^2$  for Selected Probabilities**



PROBABILITIES (OR AREAS UNDER CHI-SQUARE DISTRIBUTION CURVE ABOVE GIVEN CHI-SQUARE VALUES)

	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
<i>df</i>	<i>Values of Chi-Squared</i>									
1	0.0000	0.0002	0.0010	0.0039	0.0158	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.0100	0.0201	0.0506	0.1026	0.2107	4.6052	5.9915	7.3778	9.2104	10.5965
3	0.0717	0.1148	0.2158	0.3518	0.5844	6.2514	7.8147	9.3484	11.3449	12.8381
4	0.2070	0.2971	0.4844	0.7107	1.0636	7.7794	9.4877	11.1433	13.2767	14.8602
5	0.4118	0.5543	0.8312	1.1455	1.6103	9.2363	11.0705	12.8325	15.0863	16.7496
6	0.6757	0.8721	1.2373	1.6354	2.2041	10.6446	12.5916	14.4494	16.8119	18.5475
7	0.9893	1.2390	1.6899	2.1673	2.8331	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.3444	1.6465	2.1797	2.7326	3.4895	13.3616	15.5073	17.5345	20.0902	21.9549
9	1.7349	2.0879	2.7004	3.3251	4.1682	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.1558	2.5582	3.2470	3.9403	4.8652	15.9872	18.3070	20.4832	23.2093	25.1881
11	2.6032	3.0535	3.8157	4.5748	5.5778	17.2750	19.6752	21.9200	24.7250	26.7569
12	3.0738	3.5706	4.4038	5.2260	6.3038	18.5493	21.0261	23.3367	26.2170	28.2997
13	3.5650	4.1069	5.0087	5.8919	7.0415	19.8119	22.3620	24.7356	27.6882	29.8193
14	4.0747	4.6604	5.6287	6.5706	7.7895	21.0641	23.6848	26.1189	29.1412	31.3194
15	4.6009	5.2294	6.2621	7.2609	8.5468	22.3071	24.9958	27.4884	30.5780	32.8015
16	5.1422	5.8122	6.9077	7.9616	9.3122	23.5418	26.2962	28.8453	31.9999	34.2671
17	5.6973	6.4077	7.5642	8.6718	10.0852	24.7690	27.5871	30.1910	33.4087	35.7184
18	6.2648	7.0149	8.2307	9.3904	10.8649	25.9894	28.8693	31.5264	34.8052	37.1564
19	6.8439	7.6327	8.9065	10.1170	11.6509	27.2036	30.1435	32.8523	36.1908	38.5821
20	7.4338	8.2604	9.5908	10.8508	12.4426	28.4120	31.4104	34.1696	37.5663	39.9969
21	8.0336	8.8972	10.2829	11.5913	13.2396	29.6151	32.6706	35.4789	38.9322	41.4009
22	8.6427	9.5425	10.9823	12.3380	14.0415	30.8133	33.9245	36.7807	40.2894	42.7957
23	9.2604	10.1957	11.6885	13.0905	14.8480	32.0069	35.1725	38.0756	41.6383	44.1814
24	9.8862	10.8563	12.4011	13.8484	15.6587	33.1962	36.4150	39.3641	42.9798	45.5584
25	10.5196	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3140	46.9280
26	11.1602	12.1982	13.8439	15.3792	17.2919	35.5632	38.8851	41.9231	45.6416	48.2898
27	11.8077	12.8785	14.5734	16.1514	18.1139	36.7412	40.1133	43.1945	46.9628	49.6450
28	12.4613	13.5647	15.3079	16.9279	18.9392	37.9159	41.3372	44.4608	48.2782	50.9936
29	13.1211	14.2564	16.0471	17.7084	19.7677	39.0875	42.5569	45.7223	49.5878	52.3355
30	13.7867	14.9535	16.7908	18.4927	20.5992	40.2560	43.7730	46.9792	50.8922	53.6719