1. If $F(x) = \int_1^{e^{x^2}} \cos(\pi \ln t)\, dt$ then $F'(1) =$

(a) $-2e$

(b) 0

(c) -1

(d) 1

(e) e^2

$$F'(x) = \frac{d}{dx}\left(\int_1^{e^{x^2}} \cos(\pi \ln t)\, dt\right)$$
$$= \cos(\pi \ln e^{x^2}) \cdot 2x e^{x^2}$$
$$= 2x e^{x^2} \cos(\pi x^2)$$
$$F'(1) = 2e^1 \cos(\pi) = -2e.$$

2. $\int \sqrt{\sin x} \cos^5 x\, dx = \int \sin^{1/2} x \cos^4 x \cos x\, dx = \int \sin^{1/2} x (1-\sin^2 x)^2 \cos x\, dx$

$= \int \sin^{1/2} x (1 - 2\sin^2 x + \sin^4 x) \cos x\, dx$

(a) $\frac{2}{3}\sin^{3/2} x - \frac{4}{7}\sin^{7/2} x + \frac{2}{11}\sin^{11/2} x + c$

(b) $\sin^{3/2} x - \frac{2}{3}\sin^{7/2} x + \frac{2}{11}\sin^{11/2} x + c$

(c) $2\sin^{1/2} x - 3\sin^{7/2} x + 11\sin^{11/2} x + c$

(d) $\cos^{5/2} x - \frac{3}{2}\sin x \cos^6 x + \frac{5}{6}\cos^5 x \cos x + c$

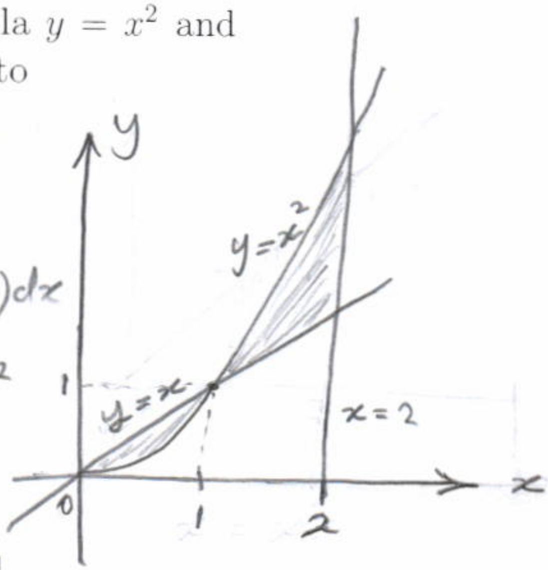
(e) $\sin^{1/2} x \cos^4 x + \sin x \cos^{5/2} x + \frac{3}{4}\cos^4 x \sin^{3/2} x + c$

$= \int (u^{1/2} - 2u^{5/2} + u^{9/2})\, du$

(Where $u = \sin x$, $du = \cos x\, dx$)

$= \frac{2}{3}u^{3/2} - \frac{4}{7}u^{7/2} + \frac{2}{11}u^{11/2} + C$, $u = \sin x$.

3. The area of the region between the parabola $y = x^2$ and the line $y = x$ from $x = 0$ to $x = 2$ is equal to



(a) 1

(b) $\frac{3}{2}$

(c) $\frac{1}{6}$

(d) 2

(e) $\frac{5}{6}$

$x = x^2 \Rightarrow x(x-1) = 0$

$\Rightarrow x = 0, 1$

$A = \int_0^1 (x - x^2)\,dx + \int_1^2 (x^2 - x)\,dx$

$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 + \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_1^2$

$= \frac{1}{2} - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2}$

$= -1 + \frac{6}{3} = 1.$

4. The sequence $a_n = \ln n - \ln(n-1)$

(a) converges to 0

(b) diverges

(c) converges to 1

(d) converges to -1

(e) converges to 5

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln n - \ln(n-1)$

$= \lim_{n\to\infty} \ln \frac{n}{n-1}$

$= \ln\left(\lim_{n\to\infty} \frac{n}{n-1}\right)$

$= \ln(1)$

$= 0.$

5. The length of the curve of $y = \frac{1}{3}(x^2+2)^{3/2}$ from $x = 0$ to $x = 3$ is equal to

(a) 12

(b) 10

(c) 8

(d) 6

(e) 4

$$\frac{dy}{dx} = \frac{1}{2}(x^2+2)^{1/2} \cdot (2x) = x\sqrt{x^2+2}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^2(x^2+2)$$

$$L = \int_0^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_0^3 \sqrt{x^4+2x^2+1}\, dx$$

$$= \int_0^3 \sqrt{(x^2+1)^2}\, dx = \int_0^3 (x^2+1)\, dx$$

$$= \left[\frac{1}{3}x^3 + x\right]_0^3 = 9+3 = 12.$$

6. The improper integral $\int_1^2 \frac{dx}{(x-1)^{4/3}}$

(a) diverges

(b) converges to 1

(c) converges to 2

(d) converges to $\sqrt[3]{2}$

(e) converges to -3

$$= \lim_{t\to 1^+} \int_t^2 (x-1)^{-4/3}\, dx$$

$$= \lim_{t\to 1^+} -3(x-1)^{-1/3}\Big]_t^2$$

$$= \lim_{t\to 1^+} -3(1)^{-1/3} + 3(t-1)^{-1/3}$$

$$= -3 + 3\lim_{t\to 1^+} \frac{1}{(t-1)^{1/3}}$$

$$= -3 + \infty = \infty.$$

The integral diverges.

7. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ is

$b_n = \frac{1}{n^2} > 0,$

$\frac{1}{(n+1)^2} < \frac{1}{n^2}$ for all $n \geq 1$

and

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n^2} = 0.$

So the series converges by the alternating series test.

(a) Convergent by the alternating series test

(b) Divergent by the alternating series test

(c) Divergent by the root test

(d) Convergent by the integral test

(e) A series where the alternating series test is inconclusive.

8. $\sum_{n=0}^{\infty} \left(\frac{3}{4^n} + \frac{1}{3^n}\right) =$

(a) $\frac{11}{2}$

(b) $\frac{3}{4}$

(c) $\frac{5}{2}$

(d) $\frac{7}{2}$

(e) 4

- $\sum_{n=0}^{\infty} \frac{3}{4^n} = 3\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$ convergent geometric series with $a = 1$, $r = \frac{1}{4}$.
  Its sum is $\frac{1}{1-1/4} = 4/3$
- Also, $\sum_{n=0}^{\infty} \frac{1}{3^n}$ is a convergent geometric series with $a = 1$, $r = \frac{1}{3}$.
  Its sum is $\frac{1}{1-1/3} = 3/2$

So $\sum_{n=0}^{\infty} \left(\frac{3}{4^n} + \frac{1}{3^n}\right) = 3\sum_{n=0}^{\infty} \frac{1}{4^n} + \sum_{n=0}^{\infty} \frac{1}{3^n}$

$= 3\left(\frac{4}{3}\right) + \frac{3}{2} = \frac{11}{2}.$

9. The series $\sum_{n=1}^{\infty} (\tan^{-1} n)^n$ is

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left|(\tan^{-1} n)^n\right|^{1/n}$$

$$= \lim_{n\to\infty} \tan^{-1} n$$

$$= \frac{\pi}{2} > 1$$

The series diverges by the root test.

(a) Divergent by the root test

(b) Convergent by the root test

(c) Convergent by the ratio test

(d) A series where the root test is inconclusive

(e) Convergent by the comparison test

10. $\int \frac{x\cos x^2 + 1 + \sin^2 x^2}{1+\sin^2 x^2}\,dx = \int\left(\frac{x\cos x^2}{1+\sin^2 x^2} + 1\right)dx$

$$= \int \frac{x\cos x^2}{1+(\sin x^2)^2}\,dx + x + c$$

$$\begin{cases} u = \sin(x^2) \\ du = 2x\cos(x^2)\,dx \end{cases}$$

$$= \frac{1}{2}\int \frac{du}{1+u^2} + x + c$$

$$= \frac{1}{2}\tan^{-1} u + x + c$$

$$= \frac{1}{2}\tan^{-1}(\sin(x^2)) + x + c.$$

(a) $\frac{1}{2}\tan^{-1}(\sin x^2) + x + c$

(b) $\tan^{-1}(\sin x^2) + x + c$

(c) $2\tan^{-1}(\sin x^2) + x + c$

(d) $-\frac{1}{2}\tan^{-1}(\sin x^2) + c$

(e) $-2\tan^{-1}(\sin x) + 3x + c$

11. $\int e^{2x} \sin x \, dx = I$. By Parts

$u = e^{2x}$ $\quad dv = \sin x \, dx$

$du = 2e^{2x} dx$ $\quad v = -\cos x$

(a) $-\frac{1}{5} e^{2x} \cos x + \frac{2}{5} e^{2x} \sin x + c$

(b) $e^{2x} \cos x + \frac{2}{5} e^{2x} \sin x + c$

(c) $\frac{1}{5} e^{2x} \sin x + \frac{2}{5} e^{2x} \cos x + c$

(d) $\frac{3}{5} e^{2x} \cos x - \frac{2}{5} e^{2x} \sin x + c$

(e) $\frac{4}{5} e^{2x} \sin x - \frac{3}{5} e^{2x} \cos x + c$

$I = -e^{2x}\cos x + 2\int e^{2x} \cos x \, dx$

$\begin{cases} u = e^{2x} & dv = \cos x \, dx \\ du = 2e^{2x} dx & v = \sin x \end{cases}$

$= -e^{2x}\cos x + 2\left(e^{2x}\sin x - 2\int e^{2x}\sin x \, dx\right)$

$= -e^{2x}\cos x + 2e^{2x}\sin x - 4I$

Thus,

$5I = -e^{2x}\cos x + 2e^{2x}\sin x$

and $I = -\frac{1}{5}e^{2x}\cos x + \frac{2}{5}e^{2x}\sin x$.

12. $\int_0^1 \frac{x\sqrt{x}}{(1+x^2\sqrt{x})^2} \, dx = \int_0^1 \frac{x^{3/2}}{(1+x^{5/2})^2} dx$ $\quad$ Let $u = 1 + x^{5/2}$

$du = \frac{5}{2} x^{3/2} dx$

(a) $\frac{1}{5}$

(b) $\frac{1}{2}$

(c) $\frac{4}{5}$

(d) $\frac{3}{2}$

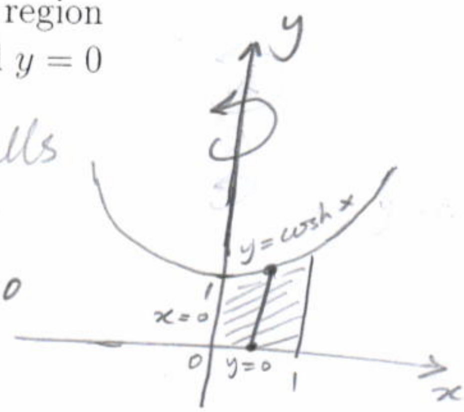
(e) 1

$= \int_1^2 \frac{2/5 \, du}{u^2} = \frac{2}{5}\int_1^2 u^{-2} du$

x	0	1
u	1	2

$= \frac{2}{5}\left[-u^{-1}\right]_1^2 = \frac{2}{5}\left(-\frac{1}{2}+1\right) = \frac{1}{5}$.

13. The volume of the solid generated by rotating the region bounded by the curves $y = \cosh x$, $x = 0$, $x = 1$ and $y = 0$ about the y-axis is given by



By cylindrical shells

radius $= r = x$

height $= h = \cosh x - 0$

$a = 0,\ b = 1$

$V = \int_a^b 2\pi r h \, dx$

$= 2\pi \int_0^1 x \cosh x \, dx$.

(a) $2\pi \int_0^1 x \cosh x \, dx$

(b) $\pi \int_0^1 x \cosh^2 x - x^2 \, dx$

(c) $2\pi \int_0^1 x^2 \cosh x \, dx$

(d) $2\pi \int_0^1 x \cosh(2x) \, dx$

(e) $\pi \int_0^1 x (\cosh^2 x - 4x^2) \, dx$

14. $\int \frac{3x^2 - 5x + 13}{(x-2)(x^2+1)} \, dx = I$. By partial fractions.

$\frac{3x^2 - 5x + 13}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx + C}{x^2+1}$

$\Rightarrow 3x^2 - 5x + 13 = A(x^2+1) + (Bx+C)(x-2)$

$\underline{x = 2}$: $12 - 10 + 13 = 5A$

$\Rightarrow \boxed{A = 3}$

$\underline{\text{coef. } x^2}$: $3 = A + B \Rightarrow \boxed{B = 0}$

$\underline{\text{Constant term}}$: $13 = A - 2C \Rightarrow \boxed{C = -5}$

$I = \int \frac{3}{x-2} dx + \int \frac{-5}{x^2+1} dx$

$= 3 \ln|x-2| - 5 \tan^{-1} x + C$

(a) $3 \ln|x-2| - 5 \tan^{-1} x + c$

(b) $3 \ln|x-2| + \ln(x^2+1) - 5 \tan^{-1} x + c$

(c) $2 \ln|x-2| + 4 \ln(x^2+1) + c$

(d) $3 \ln|x-2| + 10 \tan^{-1} x + c$

(e) $3 \ln|x-2| + 4 \ln(x^2+1) - 10 \tan^{-1} x + c$

15. The series $\sum_{n=1}^{\infty}(-1)^n \frac{10^n}{(n+1)!}$ is

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{10^{n+1}}{(n+2)!}\frac{(n+1)!}{10^n}\right|$$

$$= \lim_{n\to\infty}\frac{10}{n+2} = 0 < 1$$

The series is absolutely convergent by the ratio test.

(a) Absolutely convergent

(b) Divergent

(c) Conditionally covergent

(d) Divergent by the nth term test of divergence

(e) Convergent by the comparison test

16. $\int_0^{\sqrt{3}/2} \frac{1}{(1-x^2)^{5/2}}\,dx =$

let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$(1-x^2)^{5/2} = (1-\sin^2 x)^{5/2} = \cos^5 x$.

x	0	$\sqrt{3}/2$
θ	0	$\pi/3$

$$= \int_0^{\pi/3} \frac{\cos\theta\, d\theta}{\cos^5\theta} = \int_0^{\frac{\pi}{3}} \frac{1}{\cos^4\theta}\, d\theta$$

$$= \int_0^{\pi/3} \sec^4\theta\, d\theta = \int_0^{\pi/3} \sec^2\theta \sec^2\theta\, d\theta$$

$$= \int_0^{\pi/3} (1+\tan^2\theta)\sec^2\theta\, d\theta = \int_0^{\pi/3} (\sec^2\theta + \tan^2\theta\sec^2\theta)\, d\theta$$

$$= \left[\tan\theta + \frac{\tan^3\theta}{3}\right]_0^{\frac{\pi}{3}} = \tan\frac{\pi}{3} + \frac{1}{3}\tan^3\frac{\pi}{3} - \tan 0 - \frac{1}{3}\tan^3 0$$

$$= \sqrt{3} + \frac{1}{3}(\sqrt{3})^3 - 0 - 0 = 2\sqrt{3}.$$

(a) $2\sqrt{3}$

(b) $\frac{3\sqrt{3}}{2}$

(c) $3\sqrt{3}$

(d) $\frac{2\sqrt{3}}{3}$

(e) $\frac{\sqrt{3}}{6}$

17. The first three nonzero terms of the Maclaurin series of the function $f(x) = \ln(x^2 + x + e)$ are

$f(0) = \ln e = 1$, $f'(x) = \dfrac{2x+1}{x^2+x+e}$, $f'(0) = \dfrac{1}{e}$,

(a) $1 + \dfrac{x}{e} + \left(\dfrac{2e-1}{2e^2}\right) x^2$

$f''(x) = \dfrac{2(x^2+x+e) - (2x+1)^2}{(x^2+x+e)^2}$

(b) $1 + \dfrac{x}{e} + (2e-1)\, x^2$

$f''(0) = \dfrac{2e-1}{e^2}$

(c) $1 + \dfrac{x}{e} + \left(\dfrac{3e+1}{e^2}\right) x^2$

The first three nonzero terms of the Maclaurin series of f are.

(d) $1 + x + \dfrac{1}{e} x^2$

$f(0) + \dfrac{f'(0)}{1!} x + \dfrac{f''(0)}{2!} x^2$

(e) $1 + e^x + \left(\dfrac{2e+1}{3}\right) x^2$

$= 1 + \dfrac{1}{e} x + \dfrac{2e-1}{2e^2} x^2$.

18. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n}(x-3)^n$ is

$\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(-2)^{n+1}(x-3)^{n+1}\; n}{(n+1)\,(-2)^n (x-3)^n}\right|$

(a) $\left(\dfrac{5}{2}, \dfrac{7}{2}\right]$

$\displaystyle = \lim_{n\to\infty} \frac{2n}{n+1} |x-3| = 2|x-3|$

(b) $\left(\dfrac{5}{2}, \dfrac{7}{2}\right)$

Using the Ratio Test, the series converges if $2|x-3| < 1 \Rightarrow |x-3| < \frac{1}{2} \Rightarrow \frac{5}{2} < x < \frac{7}{2}$.

(c) $\left[\dfrac{5}{2}, \dfrac{7}{2}\right]$

$x = \frac{5}{2}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n(-\frac{1}{2})^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ divergent harmonic series.

(d) $\left(\dfrac{5}{2}, \dfrac{7}{4}\right)$

$x = \frac{7}{2}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ convergent alternating harmonic series

(e) $\left(\dfrac{5}{4}, \dfrac{7}{4}\right]$

So, the interval of convergence is $\left(\frac{5}{2}, \frac{7}{2}\right]$.