MATH102.5 (Term 142)

Quiz 1 (Sects. 5.3, 5.4, 5.5) Duration: 20 mn

Name:

ID number:

1.) (4 pts) Use Riemann sum to write the integral $\int_0^1 (x^3-x)\,dx$ as limit of sums. Find the value of this limit.

2.) (6 pts) Evaluate the integrals

$$A=\int 2x^9\sqrt{x^5+1}\,dx,\quad B=\int_0^{\frac{\pi^2}{36}} \frac{dx}{\sqrt{x}\csc(\sqrt{x}+\frac{\pi}{6})},\quad C=\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\,dx.$$

1.) $\Delta x = \frac{1}{n}$, $x_i^* = \frac{i}{n}$

$$R_n = \frac{1}{n}\sum_{i=1}^{n}\left[(x_i^*)^3 - x_i^*\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[\frac{i^3}{n^3} - \frac{i}{n}\right]$$

$$= \frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2 - \frac{1}{n^2}\,\frac{n(n+1)}{2}.$$

Thus, $\int_0^1 (x^3-x)\,dx = \lim_{n\to\infty} R_n$.

$$\lim_{n\to\infty} R_n = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}.$$

We have $\int_0^1 (x^3-x)\,dx = -1/4$.

2.) $A=\int 2x^9\sqrt{x^5+1}\,dx$

$u = x^5+1 \Rightarrow du = 5x^4\,dx$

We also have $x^5 = u-1$

Thus, $A = \int \frac{2}{5}x^5\sqrt{x^5+1}\,\underbrace{5x^4\,dx}_{du}$

$$= \int \frac{2}{5}(u-1)\sqrt{u}\,du$$

$$= \frac{2}{5}\int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{2}{5}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$\boxed{A = \frac{4}{25}(x^5+1)^{5/2} - \frac{4}{15}(x^5+1)^{3/2} + C}$$

$$B = \int_0^{\frac{\pi^2}{36}} \frac{dx}{\sqrt{x}\csc(\sqrt{x}+\pi/6)}$$

$u = \sqrt{x}+\frac{\pi}{6} \Rightarrow du = \frac{dx}{2\sqrt{x}}$

$$\Rightarrow B = 2\int_{\pi/6}^{\frac{\pi}{3}} \frac{du}{\csc u} = 2\int_{\pi/6}^{\pi/3} \sin u\,du$$

$$= 2\left[-\cos u\right]_{\pi/6}^{\pi/3} = 2\left(\cos\tfrac{\pi}{6} - \cos\tfrac{\pi}{3}\right)$$

$$\boxed{B = \sqrt{3}-1}$$

$$C = \int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$$

$u = \cos x \Rightarrow du = -\sin x\,dx$

$$\Rightarrow C = -\int_1^0 \frac{du}{1+u^2} = \left[\tan^{-1} u\right]_0^1 = \pi/4$$

$$\boxed{C = \pi/4.}$$

MATH 102.29 (Term 142)

Quiz 1 (Sects 5.3, 5.4, 5.5)

Name: Duration: 20 mn

ID number:

1.) (4 pts) Use Riemann sum to write the integral $\int_{-1}^{0}(x^2-x)\,dx$ as limit of sums. Find the value of this limit!

2.) (6 pts) Evaluate the integrals

$$A=\int_0^1 (x-1)(x+2)^{50}\,dx,\quad B=\int \frac{e^{4t}\tan e^{2t}-1}{e^{2t}}\,dt,\quad C=\int \frac{e^{\sqrt{\cos x}}}{\sqrt{\cos x}}\sin x\,dx$$

1.) $\Delta x=\frac{1}{n}$, $x_i^*=-1+\frac{i}{n}$

$$R_n=\frac{1}{n}\sum_{i=1}^{n}\left[(x_i^*)^2-(x_i^*)\right]$$

$$=\frac{1}{n}\sum_{i=1}^{n}\left[\left(-1+\frac{i}{n}\right)^2-\left(-1+\frac{i}{n}\right)\right]$$

$$=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}-\frac{3i}{n}+2\right)$$

$$=\frac{1}{n^3}\frac{n(n+1)(2n+1)}{6}-\frac{3}{n^2}\frac{n(n+1)}{2}+\frac{2}{n}\cdot n$$

Now, $\lim_{n\to\infty} R_n=\frac{1}{3}-\frac{3}{2}+2=5/6$.

and $\int_{-1}^{0}(x^2-x)\,dx=5/6$.

2.) $A=\int_0^1 (x-1)(x+2)^{50}\,dx$

$u=x+2 \Rightarrow du=dx$

We also have $x=u-2$

$$\Rightarrow A=\int_2^3 (u-3)u^{50}\,du=\int_2^3 (u^{51}-3u^{50})\,du$$

$$=\left[\frac{u^{52}}{52}-\frac{3u^{51}}{51}\cancel{+C}\right]_2^3$$

$$\boxed{A=\frac{3^{52}}{52}-\frac{3^{52}}{51}-\frac{2^{52}}{52}+3\frac{2^{51}}{51}.}$$

$$B=\int \frac{e^{4t}\tan e^{2t}-1}{e^{2t}}\,dt$$

$$=\underbrace{\int e^{2t}\tan e^{2t}\,dt}-\int e^{-2t}\,dt$$

$u=e^{2t}$, $du=2e^{2t}\,dt$

$$=\frac{1}{2}\int \tan u\,du+\frac{1}{2}e^{-2t}+C$$

$$=-\frac{1}{2}\ln|\cos u|+\frac{1}{2}e^{-2t}+C$$

$$\boxed{B=-\frac{1}{2}\ln\left|\cos e^{2t}\right|+\frac{1}{2}e^{-2t}+C.}$$

$$C=\int \frac{e^{\sqrt{\cos x}}}{\sqrt{\cos x}}\sin x\,dx$$

$u=\sqrt{\cos x}$, $du=\frac{-\sin x}{2\sqrt{\cos x}}\,dx$

$$\Rightarrow C=-2\int e^u\,du$$

$$=-2e^u+C$$

$$\boxed{C=-2e^{\sqrt{\cos x}}+C}$$