MATH 102.5 (Term 142)

Quiz 5 (Sects. 10.1, 10.2 & 10.3) Duration: 20mn

Name: ID number:

1.)(3pts)Find the limit of the sequence $\{n\sin\frac{3}{n}\}_{n=1}^{\infty}$.
2.)(3pts)What is the value of sum $\sum_{n=1}^{\infty}\frac{(-e)^{1-n}}{(\sqrt{2})^{2-2n}}$.
3.)(4pts)Do the following series converge or diverge $\sum_{n=1}^{\infty}\frac{n}{(n^2+3)^2}$, $\sum_{n=2}^{\infty}\frac{1}{n(4+(\ln n)^2)}$ (Hint: use the integral test for both series).

1) $\lim_{n\to\infty} n\sin\frac{3}{n} = \lim_{n\to\infty} 3\,\frac{\sin\frac{3}{n}}{\frac{3}{n}}$

$= 3$

2) $\sum_{n=1}^{\infty}\frac{(-e)^{1-n}}{(\sqrt{2})^{2-2n}} = \sum_{n=1}^{\infty}\frac{(-e)^{1-n}}{2^{1-n}}$

$= \sum_{n=1}^{\infty}\left(-\frac{2}{e}\right)^{n-1}$

$= \frac{1}{1+\frac{2}{e}} = \frac{e}{2+e}$

3) $f(x) = \frac{x}{(x^2+3)^2}$

$f'(x) = \frac{-3(x^2-1)}{(x^2+3)^3}$

f is positive, continuous and decreasing for $x \geq 1$

$\int_1^{\infty}\frac{x}{(x^2+3)^2}dx = \left[-\frac{1}{2}\frac{1}{x^2+3}\right]_1^{\infty}$

$= \frac{1}{8}$

$\Rightarrow$ By integral test, the series $\sum_{n=1}^{\infty}\frac{n}{(n^2+3)^2}$ converges.

$g(x) = \frac{1}{x(4+(\ln x)^2)}$

$g'(x) = -\frac{((\ln x)^2+2\ln x+4)}{x^2(4+(\ln x)^2)^2}$

$g'(x) < 0,\ \forall x \geq 1$

g is positive, continuous, decreasing for $x \geq 1$.

$\int_2^{\infty}\frac{1}{x(4+(\ln x)^2)}dx = \int_{\ln 2}^{\infty}\frac{du}{4+u^2}$

$= \frac{1}{2}\left[\tan^{-1}\left(\frac{u}{2}\right)\right]_{\ln 2}^{\infty}$

$= \frac{1}{2}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{\ln 2}{2}\right)\right]$

By the integral test, the serie $\sum_{n=2}^{\infty}\frac{1}{n(4+(\ln n)^2)}$ converges.

MATH 102.29 (Term 142)

Quiz 5 (Sects. 10.1, 10.2 & 10.3) Duration: 20mn

Name: ID number:

1.)(3pts)Find the limit of the sequence $\sqrt{2}(\sqrt{10}-\sqrt{3})$, $\sqrt{3}(\sqrt{11}-\sqrt{4})$, $\sqrt{4}(\sqrt{12}-\sqrt{5})$, $\sqrt{5}(\sqrt{13}-\sqrt{6})$,...

2.)(3pts)What is the value of sum $\sum_{n=1}^{\infty}\frac{3^{1+n}}{(\sqrt{2e})^{2+2n}}$.

3.)(4pts)Do the the following series converge or diverge $\sum_{n=2}^{\infty}\frac{n}{(n^2-2)^2}$, $\sum_{n=3}^{\infty}\frac{1}{n\ln n}$ (Hint: use the integral test for both series).

1a) $a_n = \sqrt{n}\left(\sqrt{n+8}-\sqrt{n+1}\right)$, $n=2,3,\ldots$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{\sqrt{n}\,(n+8-(n+1))}{\sqrt{n+8}+\sqrt{n+1}}$$

$$= \lim_{n\to\infty}\frac{7}{\sqrt{1+\frac{8}{n}}+\sqrt{1+\frac{1}{n}}} = \frac{7}{2}$$

2.) $$\sum_{n=1}^{\infty}\frac{3^{1+n}}{(\sqrt{2e})^{2+2n}} = \sum_{n=1}^{\infty}\frac{3^{1+n}}{(2e)^{1+n}}$$

$$= \sum_{n=1}^{\infty}\left(\frac{3}{2e}\right)^2\left(\frac{3}{2e}\right)^{n-1}$$

$$= \frac{\frac{9}{4e^2}}{1-\frac{3}{2e}} = \frac{9}{2e(2e-3)}$$

3.) $f(x) = \frac{x}{(x^2-2)^2}$

$f'(x) = -\frac{3x^2+2}{(x^2-2)^3}$

f is continuous, positive and decreasing, for $x \geq 2$

$$\int_2^{\infty}\frac{x}{(x^2-2)^2}\,dx = \left[\frac{-1}{2}\frac{1}{x^2-2}\right]_2^{\infty}$$

$$= \frac{1}{4}$$

By the integral test, the series $\sum_{n=2}^{\infty}\frac{n}{(n^2-2)^2}$ converges.

Now, let $g(x) = \frac{1}{x\ln x}$

$g'(x) = -\frac{(1+\ln x)}{x^2\ln^2 x}$

g is positive, continuous and decreasing for $x \geq 3$.

$$\int_3^{\infty}\frac{1}{x\ln x}\,dx = \left[\ln(\ln x)\right]_3^{\infty}$$

$$= \infty$$

By the integral test, the series $\sum_{n=3}^{\infty}\frac{n}{n\ln n}$ diverges