MATH 102.5 (Term 142)

Quiz 6 (Sects. 10.4, 10.5 & 10.6) Duration: 30mn

Name: ID number:

1.)(2pts)Use comparison test to show that the series $\sum_{n=2}^{\infty} \frac{\sin^4\left(\frac{1}{n^2-1}\right)}{(n+5)^{7/6}}$ converges.

2.)(2pts)Use ratio test to study the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(n+2)^{n+1}}$.

3.)(4pts)Do the series $\sum_{n=1}^{\infty} \left(\frac{1+\ln\sqrt[4]{n}}{2+\sqrt[4]{n}}\right)^n$ and $\sum_{n=1}^{\infty} \left(\frac{-1+9n^2}{n^2-n+1}\right)^n$ converge or diverge?

4.)(2pts)Find the smallest number of terms required to approximate the sum $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^4}$ so that $|\text{error}| < 0.0001$.

1.) $\sin^4\left(\frac{1}{n^2-1}\right) \leq 1$

$$\Rightarrow \frac{\sin^4\left(\frac{1}{n^2-1}\right)}{(n+5)^{7/6}} \leq \frac{1}{(n+5)^{7/6}}$$

$\sum_{n=2}^{\infty} \frac{1}{(n+5)^{7/6}}$ CV $\Longrightarrow \sum_{n=2}^{\infty} \frac{\sin^4\left(\frac{1}{n^2-1}\right)}{(n+5)^{7/6}}$ CV

2.) $a_n = \frac{n!}{(n+2)^{n+1}}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(n+3)^{n+2}} \frac{(n+2)^{n+1}}{n!}$$

$$= \frac{n+1}{n+3}\left(\frac{n+2}{n+3}\right)^{n+1}$$

$\lim_{n\to\infty} \frac{n+1}{n+3} = 1$ and

$$\lim_{n\to\infty}\left(\frac{n+2}{n+3}\right)^{n+1} = \lim_{n\to\infty} e^{(n+1)\ln\left(\frac{n+2}{n+3}\right)}$$

But, $\lim_{n\to\infty} (n+1)\ln\left(\frac{n+2}{n+3}\right) = \lim_{n\to\infty} \frac{\ln\left(\frac{n+2}{n+3}\right)}{\frac{1}{n+1}}$

$$\overset{(HR)}{=} \lim_{n\to\infty} \frac{\frac{1}{(n+3)^2}}{\frac{-1}{(n+1)^2}}$$

$$= -1$$

$\Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = e^{-1} < 1$

$\Rightarrow \sum_{n=1}^{\infty} \frac{n!}{(n+2)^{n+1}}$ CV

3.) $a_n = \left(\frac{1+\ln\sqrt[4]{n}}{2+\sqrt[4]{n}}\right)^n$, $\sqrt[n]{a_n} = \frac{1+\ln\sqrt[4]{n}}{2+\sqrt[4]{n}}$

$\lim_{n\to\infty} \sqrt[n]{a_n} = 0 < 1 \Rightarrow \sum a_n$ CV

$b_n = \left(\frac{-1+9n^2}{n^2-n+1}\right)^n$, $\sqrt[n]{b_n} = \frac{-1+9n^2}{n^2-n+1}$

$\lim_{n\to\infty} \sqrt[n]{b_n} = 9 > 1 \Rightarrow \sum b_n$ DIV

4.) $|\text{error}| < \frac{1}{(n+1)^4}$

We require $\frac{1}{(n+1)^4} < 10^{-4}$

$(n+1)^4 > 10^4$

$n+1 > 10$, $n > 9$

$n = 10$

MATH 102.29 (Term 142)

Quiz 6 (Sects. 10.4, 10.5 & 10.6) Duration: 30mn

Name: ID number:

1.)(2pts)Use comparison test to show that the series $\sum_{n=0}^{\infty} \frac{\cos^8\left(\frac{1}{n^2+1}\right)}{(n+1)^{5/4}}$ converges.

2.)(2pts)Use ratio test to study the convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)!}{n^n}$.

3.)(4pts)Do the series $\sum_{n=1}^{\infty} \left(\frac{1+3\ln\sqrt{n}}{2+n}\right)^n$ and $\sum_{n=1}^{\infty} \left(\frac{1+5n^2}{n^2+n+1}\right)^n$ converge or diverge?

4.)(2pts)Find the smallest number of terms required to approximate the sum $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^5}$ so that $|\text{error}| < 0.00001$.

---

1) $\cos^8\left(\frac{1}{n^2+1}\right) \leq 1$

$$\frac{\cos^8\left(\frac{1}{n^2+1}\right)}{(n+1)^{5/4}} \leq \frac{1}{(n+1)^{5/4}}$$

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)^{5/4}} \text{ CV} \Rightarrow \sum_{n=0}^{\infty} \frac{\cos^8\left(\frac{1}{n^2+1}\right)}{(n+1)^{5/4}} \text{ CV}$$

2) $a_n = \frac{(n+1)!}{n^n}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+2)!}{(n+1)^{n+1}} \frac{n^n}{(n+1)!}$$

$$= \frac{n+2}{n+1}\left(\frac{n}{n+1}\right)^n$$

$\lim_{n\to\infty} \frac{n+2}{n+1} = 1$, and

$$\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} e^{n\ln\left(\frac{n}{n+1}\right)}.$$

But, $\lim_{n\to\infty} n\ln\left(\frac{n}{n+1}\right) = \lim_{n\to\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}}$

$$= \lim_{n\to\infty} \frac{\frac{1}{(n+1)^2}}{-\frac{1}{n^2}}$$

$$= -1$$

$$\Rightarrow \lim \frac{a_{n+1}}{a_n} = e^{-1} < 1$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{(n+1)!}{n^n} \text{ CV.}$$

3) $a_n = \left(\frac{1+3\ln\sqrt{n}}{2+n}\right)^n$, $\sqrt[n]{a_n} = \frac{1+3\ln\sqrt{n}}{2+n}$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{1+\frac{3}{2}\ln n}{2+n} = 0 < 1$$

$$\Rightarrow \sum_{n=1}^{\infty} a_n \text{ CV}$$

$b_n = \left(\frac{1+5n^2}{n^2+n+1}\right)^n$, $\sqrt[n]{b_n} = \frac{1+5n^2}{n^2+n+1}$

$$\lim_{n\to\infty} \sqrt[n]{b_n} = 5 > 1$$

$$\Rightarrow \sum b_n \text{ DiV.}$$

4) $|\text{error}| < \frac{1}{(n+1)^5}$.

We require that

$$\frac{1}{(n+1)^5} < 10^{-5}$$

$$(n+1)^5 > 10^5$$

$$n+1 > 10$$

$$n > 9$$

$$n = 10$$