

Note: Show all your work. No credits for answers not supported by work.

Problem 1: (20 points) Find each of the following limits if it exists. Use ∞ and/or $-\infty$ when appropriate.

(a) $\lim_{x \rightarrow 1} \frac{x}{x-1}$

(b) $\lim_{x \rightarrow -\infty} \frac{3-2x-2x^3}{7-5x^3+4x^2}$

(c) $\lim_{x \rightarrow 6} \left[\frac{\sqrt{x-2}-2}{x-6} \right]$

Problem 2: (40 points)

(a) Find all values of A and B which will make the function continuous at $x = 1$.

$$f(x) = \begin{cases} \sqrt{2-x} & \text{if } x < 2 \\ 5A & \text{if } x = 2 \\ x - B & \text{if } x > 2. \end{cases}$$

(b) Use the definition of the derivative to find $f'(4)$ for the function $f(x) = \sqrt{x}$.

(c) The position function of a moving object is $s = f(t) = 3t^2 - t + 1$, where t is in seconds and s is in meters.

i. Find the average velocity over the interval $[4, 4.1]$

ii. Find the velocity at $t = 4$.

- (d) Find the rate of change of the surface area S of a sphere with radius r , when $r = 1.5 \text{ cm}$. Also find the percentage rate of change. (Note that $S = 4\pi r^2$)

Problem 3: (40 points)

- (a) If $y = (x + 1)^{2x}$, find y' at $(0,1)$

- (b) Find the slope of the line tangent to the graph of $x^2 + x \ln y + y^3 = 5$ at the point $(2,1)$.

(c) Find y'' for the function $y = 2^{2x} - \log_2 x$

(d) If $y = \sqrt{\frac{(x+1)^3(x-2)}{(2x+1)}}$, find y'