Problem 1: (40 points) Consider the function \( y = \frac{x^2}{x-2} \) with \( y' = \frac{x(x-4)}{(x-2)^2}, \quad y'' = \frac{8}{(x-2)^3} \)

a. Find all vertical, horizontal and slant asymptotes, if any exist.

b. Find the critical numbers.

c. Find intervals where the function is increasing and those where it is decreasing.

d. Find the local maximum and local minimum of the function.

e. Find intervals where the function is concave up and those where the function is concave down, also find the inflection points if any exist.

f. Sketch the graph of the function. Clearly indicate the critical numbers, extrema and inflection points on the graph.

Problem 2: (10 points) A rectangular portion of a field is to be enclosed by a fence and divided equally into three parts by two fences parallel to one pair of sides. If the total of 800 ft of fencing is to be used, find the dimensions that will maximize the fenced area, and find this maximum area.

Problem 3: (10 points) Use differentials to approximate \( \ln 1.01 \).

Problem 4: (40 points) Evaluate the integrals:

(a) \( \int_{1}^{4} \frac{e^{1+x} \sqrt{x}}{\sqrt{x}} \, dx \)

(b) \( \int (e^x + x^e) \, dx \)

(c) \( \int \frac{1}{x \ln x} \, dx \)

(d) \( D_3 \left( \int_{1}^{3} \sqrt{x^3 + x} \, dx \right) \)

(e) \( \int_{1}^{1+e^x} \, dx \)
$y = \frac{x^2}{x - 2}$