

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 132 - FINAL EXAM

Wednesday – May 20, 2015

Dr. Mohammad Z. Abu-Sbeih

TIME: 12:30 – 3:00 P.M.

Student Number:

Serial Number:

Name:

Important Notes

DO NOT USE CALCULATORS OF ANY TYPE

1. Write your serial number, student number, section number and name on both the answer sheet and question paper.
2. Show all your work. No credits given for answers not supported by work.
3. Write neatly and legibly. You may lose points for messy work.
4. Check that the exam paper has 25 different questions.

Question	Maximum Points	Student Score
1	40	
2	40	
3	40	
4	40	
5	40	
6	40	
Total	240	

(1) Consider the function $f(x) = \frac{x^2 + x}{x^2 - 1}$

(a) Find the limit if it exists. Use ∞ or $-\infty$ when appropriate.

i. $\lim_{x \rightarrow -1} \frac{x^2 + x}{x^2 - 1}$

ii. $\lim_{x \rightarrow 1} \frac{x^2 + x}{x^2 - 1}$

iii. $\lim_{x \rightarrow \infty} \frac{x^2 + x}{x^2 - 1}$

(b) Find all points of discontinuity of the function $f(x) = \frac{x^2 + x}{x^2 - 1}$ if any exists. Write the type of each one.

(c) Find the vertical, horizontal and slant asymptotes of the function $f(x) = \frac{x^2 + x}{x^2 - 1}$, if any exists.

a. Vertical asymptote:

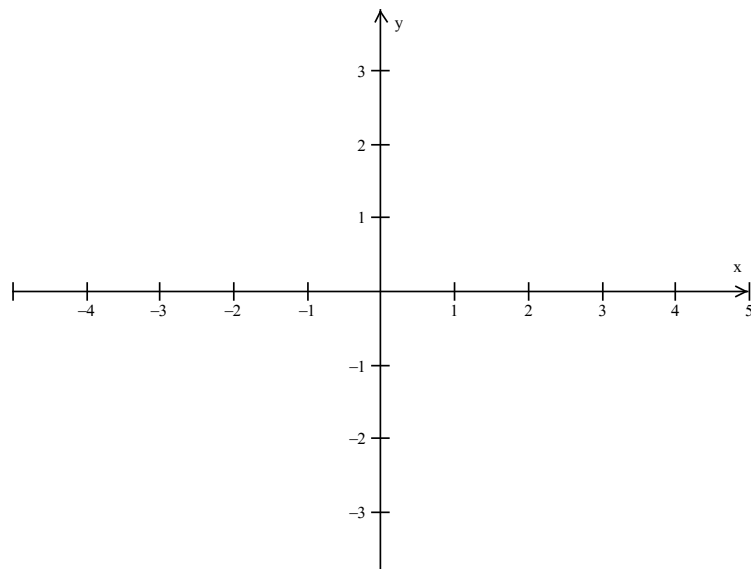
b. Horizontal asymptote:

c. Slant asymptote:

(2) For the function $f(x) = \frac{x^2 + x}{x^2 - 1}$, we have

$$f'(x) = -\frac{1}{(x-1)^2}, \quad \text{and} \quad f''(x) = \frac{2}{(x-1)^3}$$

- (a) Find the intervals where the function is increasing and those where the function is decreasing.
- (b) Find local extrema if any exists.
- (c) Find the intervals where the graph of the function is concave up and those where the graph is concave down.
- (d) Find the inflection points if any exists.
- (e) Sketch the graph of the function. Label all important points on the graph and the asymptotes (if any).



(3) (a) Find $\frac{dy}{dx}$ for each function

i. $y = x^3 + 3^x$

ii. $y = \ln(x^2 + \sin x)$

iii. $y = \tan(\sec x)$

iv. $y = (1 + 2x)^{3x}$

(b) A manufacturer wants to design a rectangular box with square bottom, having a storage capacity of 1000 cubic ft. Find the least amount of material needed to make the box.

(4) Evaluate the following integrals

(a) $\int \frac{x+2}{x+1} dx$

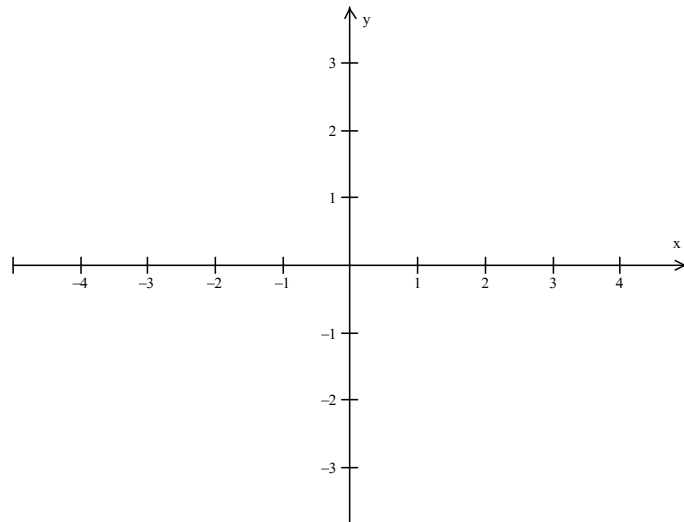
(b) $\int x \sin x dx$

(c) $\int \frac{x dx}{\sqrt{x^2+1}}$

(d) If $\int \frac{du}{[u^2 \pm a^2]^{\frac{3}{2}}} = \frac{\pm u}{a^2 \sqrt{u^2 \pm a^2}} + C$, then $\int_0^1 \frac{dx}{(x^2 + 2x + 2)^{\frac{3}{2}}}$ is equal to:

(5) (a) Given that $y'' = x^2 - 2x$, $y'(1) = 0$, $y(0) = 1$. Find y .

(b) Sketch the region bounded by $y = x^2$ and $y = x + 2$ and find its area.



(c) Using differentials to approximate $\ln 1.01$

(d) Find closest distance from the origin to the line $y = 10 + 3x$

(6) (a) Let $f(r, \theta, t) = r \sin \theta + r \cos \theta + r^2 \theta^3 t^5$

i. Find $\frac{df}{dr}(2, 0, 1)$

ii. Find $f_{r\theta t}(1, 2, 1)$

- (b) Let $f(x, y) = xy - \frac{1}{x} - \frac{1}{y}$. Find the critical points and determine whether they correspond to local maximum, to local minimum, or to neither, or whether the test fails.